Section 3.3  

The Unit Circle and Circular Functions

The unit circle is a circle with a radius of 1.

Recall that \( s = r \theta \). If \( r = 1 \), then the arclength \( s = \theta \) radians.

\( (s \) is a real number)\

Circular Function Definitions

For any real number \( s \) represented by a directed arc on the unit circle,

\[
\sin s = y \quad \cos s = x \quad \tan s = \frac{y}{x}
\]

\[
\csc s = \frac{1}{y} \quad \sec s = \frac{1}{x} \quad \cot s = \frac{x}{y}
\]

Recall that we previously defined the trigonometric functions as

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}
\]
Example 1: Given that real number \( s = \frac{-3\pi}{2} \), find

a) \( \sin s \), b) \( \cos s \), c) \( \tan s \).

\[
\sin \left( \frac{-3\pi}{2} \right) = y = 1
\]
\[
\cos \left( \frac{-3\pi}{2} \right) = x = 0
\]
\[
\tan \left( \frac{-3\pi}{2} \right) = \frac{y}{x} = \text{undefined}
\]

Example 2: Find the exact circular function values for each of the following.

a) \( \sin \frac{7\pi}{4} = \frac{-\sqrt{2}}{2} \)

b) \( \cos \frac{11\pi}{6} = \frac{\sqrt{3}}{2} \)

c) \( \tan \frac{13\pi}{3} = \frac{\sqrt{3}}{2} \)

d) \( \cot \frac{5\pi}{6} = \frac{-\sqrt{3}}{2} \)

e) \( \csc \left( \frac{-2\pi}{3} \right) = \frac{1}{\sqrt{3}} \)

f) \( \sec \frac{3\pi}{4} = \frac{1}{\frac{-\sqrt{2}}{2}} = \frac{-\sqrt{2}}{2} \)
Example 3: Find a calculator approximation for each circular function value. *Use Radian mode

a) \( \sin (0.6109) = 0.5736 \)

b) \( \cos (-1.1519) = 0.4068 \)

c) \( \sec (2.8440) = -1.0460 \)

Example 4: Find the value of \( s \) in the interval \([0, \frac{\pi}{2}]\) that makes each statement true. *Use Radian mode

a) \( \tan s = 0.2126 \)

\[ \tan^{-1}(0.2126) = s = 0.2095 \text{ Radians} \]

b) \( \csc s = 1.0219 \)

\[ \sin s = \frac{1}{1.0219} \]

\[ \sin^{-1}(1.0219) = s = 1.3634 \text{ Radians} \]

Example 5: Find the exact value of \( s \) in the given interval that has the given circular function value. Do not use a calculator.

a) \( \left[ \frac{\pi}{2}, \pi \right] ; \cos s = -\frac{1}{2} \)

\[ s = \frac{2\pi}{3} \]

b) \( \left[ \pi, \frac{3\pi}{2} \right] ; \tan s = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \)

\[ s = \frac{7\pi}{6} \]

c) \( \left[ \frac{3\pi}{2}, 2\pi \right] ; \sin s = -\frac{\sqrt{2}}{2} \)

\[ s = \frac{7\pi}{4} \]
Example 6: Find the exact value of \( s \) in the given interval that has the given circular function value. Do not use a calculator.

a) \([0, 2\pi)\); \( \sin s = -\frac{\sqrt{3}}{2} \)

\[ s = \frac{4\pi}{3}, \frac{5\pi}{3} \]

b) \([0, 2\pi)\); \( \sqrt{\cos^2 s} = \frac{1}{\sqrt{2}} \)

\[ \cos s = \pm \frac{\sqrt{2}}{2} \]

\[ s = \frac{3\pi}{4}, \frac{5\pi}{4} \]

c) \([0, 2\pi)\); \( \tan^2 s = 3 \)

\[ \tan s = \pm \sqrt{3} \]

\[ s = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \]