I ① $5A+3C = 5\begin{bmatrix}1&1&-1\\1&0&-2\end{bmatrix} + 3\begin{bmatrix}0&1\\1&-1\\2&0\end{bmatrix}$

$= \begin{bmatrix}5&5&-5\\5&0&-10\end{bmatrix} + \begin{bmatrix}0&3\\3&-3\\6&0\end{bmatrix}$

$\boxed{\text{CANNOT ADD AS DIMENSIONS DO NOT MATCH}}$

② $A \cdot H = \underbrace{\begin{bmatrix}1&1&-1\\1&0&-2\end{bmatrix}}_{2\times 3} \cdot \underbrace{\begin{bmatrix}3&-3&1\\-2&2&-1\\-4&0&-2\end{bmatrix}}_{3\times 3} = \boxed{\begin{bmatrix}5&-1&2\\11&-3&5\end{bmatrix}}$

ok

③ $D \cdot F = \underbrace{\begin{bmatrix}1&-1&1\\0&-2&1\\-2&-3&0\end{bmatrix}}_{3\times 3} \cdot \underbrace{\begin{bmatrix}1\\0\\1\end{bmatrix}}_{3\times 1} = \boxed{\begin{bmatrix}2\\1\\-2\end{bmatrix}}$

✓

④ $E \cdot (3J+2G) = \begin{bmatrix}1&0\\1&1\end{bmatrix} \cdot \left(3\begin{bmatrix}0\\-1\end{bmatrix} + 2\begin{bmatrix}2\\1\end{bmatrix}\right)$

$= \begin{bmatrix}1&0\\1&1\end{bmatrix} \cdot \left(\begin{bmatrix}0\\-3\end{bmatrix} + \begin{bmatrix}4\\2\end{bmatrix}\right)$

$= \underset{2\times 2}{\begin{bmatrix}1&0\\1&1\end{bmatrix}} \cdot \underset{2\times 1}{\begin{bmatrix}4\\-1\end{bmatrix}}$

$= \boxed{\begin{bmatrix}4\\3\end{bmatrix}}$

#5. $H \cdot D = \begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10 & -4 \end{bmatrix}}$

$3\times3 \quad 3\times3$

$2-4+3$

#6. $B - E = \begin{bmatrix} 4 & 9 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 9 \\ 0 & 1 \end{bmatrix}}$

#7. $\left[\begin{array}{cc|cc} 4 & 9 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow{S R_1 R_2} \left[\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 4 & 9 & 1 & 0 \end{array}\right] \xrightarrow{-4 \cdot R_1 + R_2} \left[\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & -4 \end{array}\right]$

$\xrightarrow{-2 \cdot R_2 + R_1} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 9 \\ 0 & 1 & 1 & -4 \end{array}\right] \rightarrow B^{-1} = \boxed{\begin{bmatrix} -2 & 9 \\ 1 & -4 \end{bmatrix}}$

#8. C^{-1} DOES NOT EXIST SINCE IT IS NOT A SQUARE MATRIX

#9. $D \cdot H = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -5 & \\ & & \\ & & \end{bmatrix}$

$\therefore$ NOT THE IDENTITY

$\therefore$ NOT INVERSES

II

(10) $-3(5x_1 + x_2 = 9)$
$(2x_1 + 3x_2 = 1)$ $\Rightarrow$

$$\begin{aligned} -15x_1 - \cancel{3x_2} &= -27 \\ 2x_1 + \cancel{3x_2} &= 1 \\ \hline \Rightarrow -13x_1 \quad &= -26 \\ \Rightarrow x_1 &= 2 \end{aligned}$$

so
$5(2) + x_2 = 9$
$\Rightarrow 10 + x_2 = 9$
$\Rightarrow x_2 = -1$
$\Rightarrow \boxed{(2, -1)}$

OTHER METHODS ARE POSSIBLE

(11) $2(3x_1 - 2x_2 = -6)$
$-6x_1 + 4x_2 = 1$ $\Rightarrow$

$$\begin{aligned} 6x_1 - 4x_2 &= -12 \\ -6x_1 + 4x_2 &= 1 \\ \hline 0 &= -11 \end{aligned}$$

$\Rightarrow$ FALSE
$\Rightarrow$ $\boxed{\text{NO SOLUTION}}$
$\therefore$ $\boxed{\text{THE LINES ARE PARALLEL}}$

(12). $5x_1 + 9x_2 = 0$
$x_1 + 2x_2 = 2$

$$\Rightarrow \left[\begin{array}{cc|c} 5 & 9 & 0 \\ 1 & 2 & 2 \end{array}\right] \xrightarrow{S R_1 R_2} \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 5 & 9 & 0 \end{array}\right] \xrightarrow{-5 \cdot R_1 + R_2} \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & -1 & -10 \end{array}\right] \longrightarrow$$

$$\xrightarrow{-1 \cdot R_2} \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 1 & 10 \end{array}\right] \xrightarrow{-2 \cdot R_2 + R_1} \left[\begin{array}{cc|c} 1 & 0 & -18 \\ 0 & 1 & 10 \end{array}\right] \Rightarrow \boxed{(-18, 10)}$$

~~13.~~

(14). $$\left[\begin{array}{cccc|c} 3 & 7 & 1 & 1 & 10 \\ 1 & 2 & 2 & 1 & 20 \end{array}\right] \xrightarrow{S R_1 R_2} \left[\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 20 \\ 3 & 7 & 1 & 1 & 10 \end{array}\right] \longrightarrow$$

$$\xrightarrow{-3 \cdot R_1 + R_2} \left[\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 20 \\ 0 & 1 & -5 & -2 & -50 \end{array}\right] \xrightarrow{-2 \cdot R_2 + R_1} \left[\begin{array}{cccc|c} 1 & 0 & 12 & 5 & 120 \\ 0 & 1 & -5 & -2 & -50 \end{array}\right]$$

$$\Rightarrow \begin{aligned} x_1 + 12x_3 + 5x_4 &= 120 \\ x_2 - 5x_3 - 2x_4 &= -50 \end{aligned} \Rightarrow \boxed{\begin{aligned} x_1 &= -12t_1 - 5t_2 + 120 \\ x_2 &= 5t_1 + 2t_2 - 50 \\ x_3 &= t_1 \\ x_4 &= t_2 \end{aligned}}$$

III. 15.

$X = A^{-1} \cdot C$

$$= \begin{bmatrix} -1 & -1 & -1 \\ 4 & 5 & 0 \\ 0 & 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \\ -5 \end{bmatrix} \Rightarrow \boxed{(-3, 5, -5)}$$

16.

$X = A^{-1} \cdot C$

$$= \begin{bmatrix} -1 & -1 & -1 \\ 4 & 5 & 0 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ -15 \end{bmatrix} \Rightarrow \boxed{(-4, -4, -15)}$$