SOLUTIONS
PRACTICE EXAM #1 ne

(I)

① $5A+3C = 5\begin{bmatrix} 1 & -1 & 4 \\ 2 & 0 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & -5 & 20 \\ 10 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 0 \\ 3 & -6 & 3 \end{bmatrix}$

$= \boxed{\begin{bmatrix} 8 & -2 & 20 \\ 13 & -6 & 8 \end{bmatrix}}$

② $A \cdot H = \begin{bmatrix} 1 & -1 & 4 \\ 2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & -11 & 7 \\ -3 & 2 & 3 \end{bmatrix}}$

2×3 ok 3×3 → 2×3

③ $D \cdot F = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 0 \\ -6 \end{bmatrix}}$

3×3 ok 3×1

④ $(3J+E)\cdot G = \left(3\begin{bmatrix} 4 & -11 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -15 \\ -1 & 4 \end{bmatrix}\right) \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$= \left(\begin{bmatrix} 12 & -33 \\ -3 & 9 \end{bmatrix} + \begin{bmatrix} 4 & -15 \\ -1 & 4 \end{bmatrix}\right) \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 16 & -48 \\ -4 & 36 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \boxed{\begin{bmatrix} -128 \\ 104 \end{bmatrix}}$

2×2 → 2×1

sorry the numbers got so big

⑤. $H \cdot D = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 12 \\ 0 & 0 & -5 \end{bmatrix}}$ pg 2 no

⑥ $B - E = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 4 & -15 \\ -1 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & 22 \\ 2 & -3 \end{bmatrix}}$

⑦. $5B - 2E = 5\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 4 & -15 \\ -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 15 & 35 \\ 5 & 10 \end{bmatrix} - \begin{bmatrix} 8 & -30 \\ 2 & 8 \end{bmatrix}$

$= \boxed{\begin{bmatrix} 7 & 65 \\ 7 & 2 \end{bmatrix}}$

⑧. $B \cdot G = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \boxed{\begin{bmatrix} 24 \\ 7 \end{bmatrix}}$

$2 \times 2 \quad 2 \times 1$

⑨. $F \cdot C = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$

$3 \times 1 \quad$ NO $\quad 2 \times 3$

CANNOT DO SINCE THE MIDDLE NUMBERS DON'T MATCH

⑩. $\left[\begin{array}{cc|cc} 3 & 7 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \xrightarrow{S R_1 R_2} \left[\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 3 & 7 & 1 & 0 \end{array}\right] \xrightarrow{-3 R_1 + R_2} \left[\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & -3 \end{array}\right]$

$\xrightarrow{-2 \cdot R_2 + R_1} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 7 \\ 0 & 1 & 1 & -3 \end{array}\right] \Rightarrow B^{-1} = \boxed{\begin{bmatrix} -2 & 7 \\ 1 & -3 \end{bmatrix}}$

(11). $\left[\begin{array}{cc|cc} 4 & -15 & 1 & 0 \\ -1 & 4 & 0 & 1 \end{array}\right] \xrightarrow{5R_1+R_2} \left[\begin{array}{cc|cc} -1 & 4 & 0 & 1 \\ 4 & -15 & 1 & 0 \end{array}\right] \xrightarrow{-1\cdot R_1} \left[\begin{array}{cc|cc} 1 & -4 & 0 & -1 \\ 4 & -15 & 1 & 0 \end{array}\right]$

$\xrightarrow{-4\cdot R_1+R_2} \left[\begin{array}{cc|cc} 1 & -4 & 0 & -1 \\ 0 & 1 & 1 & 4 \end{array}\right] \xrightarrow{4\cdot R_2+R_1} \left[\begin{array}{cc|cc} 1 & 0 & 4 & 15 \\ 0 & 1 & 1 & 4 \end{array}\right]$ so $\boxed{E^{-1} = \begin{bmatrix} 4 & 15 \\ 1 & 4 \end{bmatrix}}$

(12). $\left[\begin{array}{cc|cc} 4 & -11 & 1 & 0 \\ -1 & 3 & 0 & 1 \end{array}\right] \xrightarrow{5R_1+R_2} \left[\begin{array}{cc|cc} -1 & 3 & 0 & 1 \\ 4 & -11 & * & * \end{array}\right] \xrightarrow{-1\cdot R_1} \left[\begin{array}{cc|cc} 1 & -3 & 0 & -1 \\ 4 & -11 & 1 & 0 \end{array}\right]$

$\xrightarrow{-4\cdot R_1+R_2} \left[\begin{array}{cc|cc} 1 & -3 & 0 & -1 \\ 0 & 1 & 1 & 4 \end{array}\right] \xrightarrow{3\cdot R_2+R_1} \left[\begin{array}{cc|cc} 1 & 0 & 3 & 11 \\ 0 & 1 & 1 & 4 \end{array}\right]$ so $\boxed{J^{-1} = \begin{bmatrix} 3 & 11 \\ 1 & 4 \end{bmatrix}}$

(13) $D \cdot H = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 0 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -6 & 12 & -5 \end{bmatrix}$

3x3 · 3x3

↑ may stop here:

they ARE NOT INVERSES

---

(14) $3x_1 + x_2 = 10$

$x_1 - 4x_2 = -1 \rightarrow x_1 = 4x_2 - 1$

so $3(4x_2 - 1) + x_2 = 10$

$\Rightarrow 12x_2 - 3 + x_2 = 10$

$\Rightarrow 13x_2 = 13$

$\Rightarrow x_2 = 1$

so $x_1 = 4(1) - 1$

$= 4 - 1$

$= 3$

$\Rightarrow \boxed{(3, 1)}$

(15) $3(2x_1 + 9x_2 = 7) \Rightarrow 6x_1 + 27x_2 = 21$

$-2(3x_1 + 5x_2 = 2) \Rightarrow -6x_1 - 10x_2 = -4$

$\Rightarrow 17x_2 = 17$

$\Rightarrow x_2 = 1$

so $2x_1 + 9(1) = 7$

$\Rightarrow 2x_1 = -2$

$\Rightarrow x_1 = -1$

$\Rightarrow \boxed{(-1, 1)}$

(16) $2x_1 - 4x_2 = -2$
$-x_1 + 2x_2 = 1$

$\Rightarrow \left[\begin{array}{cc|c} 2 & -4 & -2 \\ -1 & 2 & 1 \end{array}\right] \xrightarrow{5R_1 R_2} \xrightarrow{-1 \cdot R_1} \left[\begin{array}{cc|c} 1 & -2 & -1 \\ 2 & -4 & 2 \end{array}\right]$

$\xrightarrow{-2 \cdot R_1 + R_2} \left[\begin{array}{cc|c} 1 & -2 & -1 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow$ ALL REAL NUMBERS

entire row is zeros

(17) $3x_1 + x_2 = 3$
$5x_1 + 2x_2 = 4$

$\Rightarrow \left[\begin{array}{cc|c} 3 & 1 & 3 \\ 5 & 2 & 4 \end{array}\right] \xrightarrow{-2 \cdot R_1 + R_2} \left[\begin{array}{cc|c} 3 & 1 & 3 \\ -1 & 0 & -2 \end{array}\right]$

$\xrightarrow{5R_1 R_2} \left[\begin{array}{cc|c} -1 & 0 & -2 \\ 3 & 1 & 3 \end{array}\right] \xrightarrow{-1 \cdot R_1} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 3 & 1 & 3 \end{array}\right] \xrightarrow{-3 \cdot R_1 + R_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \end{array}\right]$

$\Rightarrow x_1 = 2$
$x_2 = -3$ $\Rightarrow \boxed{(2, -3)}$

(18) $x_1 + 2x_2 + 3x_3 = 10$
$5x_1 + 9x_2 + 7x_3 = 20$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 5 & 9 & 7 & 20 \end{array}\right] \xrightarrow{-5 \cdot R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -1 & -8 & -30 \end{array}\right]$

$\xrightarrow{-1 \cdot R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & 1 & 8 & 30 \end{array}\right] \xrightarrow{-2 \cdot R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & -13 & -50 \\ 0 & 1 & 8 & 30 \end{array}\right]$

$\Rightarrow x_1 - 13x_3 = -50 \Rightarrow x_1 = 13x_3 - 50$
$x_2 + 8x_3 = 30 \quad\quad x_2 = -8x_3 + 30$

so: $\boxed{\begin{array}{l} x_1 = 13t - 50 \\ x_2 = -8t + 30 \\ x_3 = t \end{array}}$

(19) $3x_1 + x_2 + 5x_3 + 7x_4 = 5$

$2x_1 + x_2 + x_3 + x_4 = 2$

$$\Rightarrow \left[\begin{array}{cccc|c} 3 & 1 & 5 & 7 & 5 \\ 2 & 1 & 1 & 1 & 2 \end{array}\right] \xrightarrow{-1\cdot R_2 + R_1} \left[\begin{array}{cccc|c} 1 & 0 & 4 & 6 & 3 \\ 2 & 1 & 1 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{-2\cdot R_1 + R_2} \left[\begin{array}{cccc|c} 1 & 0 & 4 & 6 & 3 \\ 0 & 1 & -7 & -11 & -4 \end{array}\right]$$

$\Rightarrow x_1 + 4x_3 + 6x_4 = 3$

$x_2 - 7x_3 - 11x_4 = -4$ $\Rightarrow$

$x_1 = -4t_1 - 6t_2 + 3$

$x_2 = 7t_1 + 11t_2 - 4$

$x_3 = t_1$

$x_4 = t_2$

(20) $X = A^{-1} \cdot C$

$$= \begin{bmatrix} -5 & -2 & -2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ -1 \end{bmatrix} \Rightarrow \boxed{(6, -3, -1)}$$

(21) $X = A^{-1} \cdot C$

$$= \begin{bmatrix} -5 & -2 & -2 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -7 \\ 5 \\ -1 \end{bmatrix} \Rightarrow \boxed{(-7, 5, -1)}$$