MATH 1324 EXAM #2wo PRINTNAMENEATLY REGIS
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*** * * PHONES OFF!! * * ***

IMPORTANT: IF YOU DO NOT SHOW YOUR WORK IN A NEAT AND ORDERLY FASHION, YOU FORFEIT YOUR CLAIM TO ANY CREDIT.

I. THE FOLLOWING ARE FINAL MATRICES FROM LINEAR PROGRAMMING PROBLEMS. READ THE SOLUTIONS FROM EACH MATRIX. YOU NEED <u>NOT</u> GIVE THE VALUES OF THE SLACK VARIABLES. **ASSUME THAT PROBLEMS #1, #2, AND #3 ARE MAXIMIZE PROBLEMS. PROBLEMS #4, #5, AND #6 ARE MINIMIZE PROBLEMS.**

1.
$$\begin{array}{c}
\begin{array}{cccccccc} x_1 & x_2 & s_1 & s_2 & s_3 & s_4 & P & K \end{array} \\
\begin{bmatrix} 0 & 0 & 4 & 4 & 1 & 0 & 0 & 42 \\ 0 & 1 & 6 & 5 & 0 & 0 & 0 & 25 \\ 1 & 0 & 8 & 5 & 0 & 0 & 0 & 32 \\ 0 & 0 & 3 & 5 & 0 & 1 & 0 & 18 \\ 0 & 0 & 8 & 3 & 0 & 0 & 1 & 600 \end{bmatrix}
\end{array}$$
(Maximize)

$x_1 = 32$
$x_2 = 25$
$P = 600$

2.
$$\begin{array}{c}
\begin{array}{ccccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & s_1 & s_2 & P & K \end{array} \\
\begin{bmatrix} 3 & 1 & 9 & 0 & 2 & 10 & 7 & 0 & 200 \\ 5 & 0 & -5 & 1 & 4 & 8 & 4 & 0 & 350 \\ 8 & 0 & 2 & 0 & 7 & 3 & 1 & 1 & 1100 \end{bmatrix} \\
\begin{array}{ccccccccc} 0 & & 0 & & 0 & & & & \end{array}
\end{array}$$
(Maximize)

$x_1 = 0$ $x_4 = 350$
$x_2 = 200$ $x_5 = 0$
$x_3 = 0$ $P = 1100$

3.
$$\begin{array}{c}
\begin{array}{ccccccc} x_1 & x_2 & x_3 & s_1 & s_2 & P & K \end{array} \\
\begin{bmatrix} 0 & 1 & 1 & 6 & 16 & 0 & 16 \\ 1 & 1 & 0 & 3 & -8 & 0 & 28 \\ 0 & 0 & 0 & 4 & 4 & 1 & 50 \end{bmatrix} \\
\begin{array}{ccccccc} & 0 & & & & & \end{array}
\end{array}$$
(Maximize)

$x_1 = 28$ $P = 50$
$x_2 = 0$
$x_3 = 16$

4.
$$\begin{array}{c}
\begin{array}{cccccccc} y_1 & y_2 & y_3 & s_1 & s_2 & s_3 & C & K \end{array} \\
\begin{bmatrix} 8 & 0 & 1 & 9 & 0 & 3 & 0 & 65 \\ 1 & 0 & 0 & 2 & 1 & 3 & 0 & 80 \\ 0 & 1 & 0 & 5 & 0 & 2 & 0 & 45 \\ 5 & 0 & 0 & 4 & 0 & 2 & 1 & 150 \end{bmatrix}
\end{array}$$
(Minimize)

$x_1 = 4$
$x_2 = 0$
$x_3 = 2$
$C = 150$

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5. $$\begin{array}{c} \begin{array}{ccccccc} y_1 & y_2 & y_3 & s_1 & s_2 & C & k \end{array} \\ \left[\begin{array}{cccccc|c} 5 & 6 & 1 & 0 & 6 & 0 & 800 \\ 2 & 4 & 0 & 1 & 3 & 0 & 400 \\ 4 & 7 & 0 & 0 & 7 & 1 & 300 \end{array}\right] \end{array}$$

(Minimize)

$x_1 = 0$

$x_2 = 7$

$C = 300$

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6. $$\begin{array}{c} \begin{array}{cccccccc} y_1 & y_2 & s_1 & s_2 & s_3 & s_4 & C & k \end{array} \\ \left[\begin{array}{ccccccc|c} 0 & 0 & 4 & 4 & 1 & 0 & 0 & 42 \\ 0 & 1 & 6 & 5 & 0 & 0 & 0 & 25 \\ 1 & 0 & 8 & 5 & 0 & 0 & 0 & 32 \\ 0 & 0 & 3 & 5 & 0 & 1 & 0 & 18 \\ 0 & 0 & 8 & 3 & 0 & 0 & 1 & 600 \end{array}\right] \end{array}$$

(Minimize)

$x_1 = 8$

$x_2 = 3$

$x_3 = 0$

$x_4 = 0$

$C = 600$

II. WE JOIN THESE PROBLEMS ALREADY IN PROGRESS. FOR EACH MATRIX, DETERMINE THE PIVOT ROW AND COLUMN. SHOW ALL YOUR COMPUTATIONS.

7. $$\begin{bmatrix} 0 & 1 & 6 & 0 & 2 & 0 & 0 & 30 \\ 1 & 0 & 3 & 0 & 8 & 0 & 0 & 24 \\ 0 & 0 & 11 & 1 & 6 & 0 & 0 & 66 \\ 0 & 0 & 10 & 0 & 5 & 1 & 0 & 85 \\ 0 & 0 & -15 & 0 & -7 & 0 & 1 & 112 \end{bmatrix}$$

ROW# 1

COL# 3

Reciprocal: 1/6

RATIOS:

$\frac{30}{6} = 5$ Least positive

$\frac{24}{3} = 8$

$\frac{66}{11} = 6$

$\frac{85}{10} = 10.5$

MUST NEGATIVE

8. $$\begin{bmatrix} 24 & 2 & 4 & 12 & 1 & 0 & 0 & 48 \\ 2 & 4 & 5 & 10 & 0 & 1 & 0 & 20 \\ -6 & -12 & -3 & -11 & 0 & 0 & 1 & 0 \end{bmatrix}$$

ROW# 2

COL# 2

Reciprocal: 1/4

RATIOS:

$\frac{48}{2} = 24$

$\frac{20}{4} = 5$

9. $$\begin{bmatrix} 1 & 0 & 22 & \frac{11}{4} & 0 & 0 & \frac{33}{2} \\ 0 & 1 & \frac{1}{6} & \frac{1}{6} & 0 & 0 & \frac{13}{6} \\ 0 & 0 & 10 & 5 & 1 & 0 & \frac{20}{3} \\ 0 & 0 & -3 & -5 & 0 & 1 & \frac{45}{4} \end{bmatrix}$$

ROW# 3

COL# 4

RECIPROCAL = 1/5

RATIOS:

$\frac{\frac{33}{2}}{\frac{11}{4}} = \frac{33}{2} \cdot \frac{4}{11} = 6$

$\frac{\frac{13}{6}}{\frac{1}{6}} = \frac{13}{6} \cdot \frac{6}{1} = 13$

$\frac{\frac{20}{3}}{\frac{5}{1}} = \frac{20}{3} \cdot \frac{1}{5} = \frac{4}{3} \approx 1.3$

III. FOR EACH OF THE FOLLOWING FIND THE INITIAL SIMPLEX TABLE. YOU NEED **NOT** FIND PIVOT POSITIONS, SOLVE, ETC. SIMPLY GENERATE THE INITIAL MATRIX, WHICH WOULD BE READY FOR THE COMPUTER.

10. MINIMIZE $C = 30x_1 + 42x_2 + 16_3$

SUBJECT TO:

$$3x_1 + 4x_2 + x_3 \geq 15$$
$$x_1 + 5x_2 + 2x_3 \geq 24$$
$$4x_1 + x_2 + 8x_3 \geq 10$$
$$x_i \geq 0$$

$$\left[\begin{array}{ccc|c} 3 & 4 & 1 & 15 \\ 1 & 5 & 2 & 24 \\ 4 & 1 & 8 & 10 \\ \hline 30 & 42 & 16 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 3 & 1 & 4 & 30 \\ 4 & 5 & 1 & 42 \\ 1 & 2 & 8 & 16 \\ \hline 15 & 24 & 10 & 0 \end{array}\right] \Rightarrow \boxed{\left[\begin{array}{ccccccc|c} 3 & 1 & 4 & 1 & 0 & 0 & 0 & 30 \\ 4 & 5 & 1 & 0 & 1 & 0 & 0 & 42 \\ 1 & 2 & 8 & 0 & 0 & 1 & 0 & 16 \\ \hline -15 & -24 & -10 & 0 & 0 & 0 & 1 & 0 \end{array}\right]}$$

11. MAXIMIZE $P = 55x_1 + 11x_2$

SUBJECT TO:

$$\left.\begin{aligned} 2x_1 + 6x_2 &\leq 35 \\ 6x_1 + 2x_2 &\leq 53 \\ 7x_1 + 9x_2 &\leq 17 \\ 12x_1 + x_2 &\leq 22 \end{aligned}\right\} \text{4 slack variables}$$
$$x_i \geq 0$$

$$\boxed{\begin{bmatrix} 2 & 6 & 1 & 0 & 0 & 0 & 0 & 35 \\ 6 & 2 & 0 & 1 & 0 & 0 & 0 & 53 \\ 7 & 9 & 0 & 0 & 1 & 0 & 0 & 17 \\ 12 & 1 & 0 & 0 & 0 & 1 & 0 & 22 \\ -55 & -11 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}$$

12. A FARMER CAN USE TWO TYPES OF PLANT FOOD, MIX A AND MIX B. THE AMOUNTS (IN KILOGRAMS) OF NITROGEN, PHOSPHORIC ACID ~~AND POTASH~~ IN EACH BAG OF MIX A ARE 9 AND 12 AND OF MIX B ARE 7 AND 18. SOIL TESTS INDICATE THAT THE FIELD NEEDS AT LEAST 350 KILOGRAMS OF NITROGEN, ~~AT LEAST~~ [redacted] AND AT LEAST 400 KILOGRAMS OF PHOSPHORIC ACID. THE COST OF EACH BAG OF MIX A IS $40 AND EACH BAG OF MIX B IS $60. SUPPOSE YOU WISH TO FIND THE NUMBER OF BAGS OF EACH MIX NEEDED TO MEET THE REQUIREMENTS WHILE MINIMIZING THE COST. GENERATE THE <u>INITIAL MATRIX</u> YOU WOULD USE TO DO THIS.

Let

$X_1 = \#$ Bags Mix A

$X_2 = \#$ Bags Mix B

NOTE: "The VARIABLES determine the colums"

"At least"

$$\Rightarrow \quad \overbrace{\begin{array}{cc} \text{MIX A} & \text{MIX B} \\ X_1 & X_2 \end{array}}$$

NIT	$9X_1$	$+$	$7X_2$	≥ 350
PHOS AC	$12X_1$	$+$	$18X_2$	≥ 400
$C =$	$40X_1$	$+$	$60X_2$	

PRE-INITIAL MATRIX

$$\Rightarrow \left[\begin{array}{cc|c} 9 & 7 & 350 \\ 12 & 18 & 400 \\ \hline 40 & 60 & 0 \end{array}\right] \xrightarrow[\text{POSE}]{\text{TRANS-}} \left[\begin{array}{cc|c} 9 & 12 & 40 \\ 7 & 18 & 60 \\ \hline 350 & 400 & 0 \end{array}\right] \Big\} \text{ 2 slack variables}$$

$$\Rightarrow \boxed{\left[\begin{array}{ccccc|c} 9 & 12 & 1 & 0 & 0 & 40 \\ 7 & 18 & 0 & 1 & 0 & 60 \\ \hline -350 & -400 & 0 & 0 & 1 & 0 \end{array}\right]}$$

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I. SOLVE THE FOLLOWING LINEAR PROGRAMMING PROBLEMS, USING MAPLE, AS DEMONSTRATED IN CLASS.

MAXIMIZE $P = 150x_1 + 240x_2 + 180x_3$

SUBJECT TO:

$$\left.\begin{aligned} x_1 + x_2 + 2x_3 &\le 32 \\ 3x_1 + 4x_2 + 2x_3 &\le 84 \end{aligned}\right\} \to \text{2 slack Variables}$$

$x_i \ge 0$

FOR THIS PROBLEM, PROVIDE THE FOLLOWING INFORMATION:

1. THE INITIAL SIMPLEX MATRIX

$$\left[\begin{array}{cccccc|c} 1 & 1 & 2 & 1 & 0 & 0 & 32 \\ 3 & 4 & 2 & 0 & 1 & 0 & 84 \\ \hline -150 & -240 & -180 & 0 & 0 & 1 & 0 \end{array}\right]$$

2. THE DETAILS OF ALL THE "mulrow" AND "pivot" COMMANDS YOU USE:

a2:=mulrow(a1,2,1/4);
a3:=pivot(a2,2,2);
a4:=mulrow(a3,1,2/3);
a5:=pivot(a4,1,3);

3. MAXIMUM FOR P: 5480

$x_1 =$ 0 $x_2 =$ 52/3 $x_3 =$ 22/3

II. A DIETICIAN IS TO ARRANGE A DIET SCHEDULE USING THREE TYPES OF FOODS AND TAKING INTO ACCOUNT THREE NUTRIENTS, AS WELL AS THE COST. EACH SERVING OF FOOD "L" CONTAINS 20 UNITS OF IRON, 10 UNITS OF VITAMIN A, AND 10 UNITS OF CALCIUM. EACH SERVING OF FOOD "M" CONTAINS 10 UNITS OF IRON, 10 UNITS OF VITAMIN A AND 15 UNITS OF CALCIUM. EACH UNIT OF FOOD "N" CONTAINS 10 UNITS OF IRON, 10 UNITS OF VITAMIN A AND 10 UNITS OF CALCIUM. THE COST PER SERVING OF THE FOOD TYPES IS 20¢ FOR TYPE "L", 24¢ FOR TYPE "M" AND 18¢ FOR TYPE "N". ASSUMING THE MINIMUM DAILY REQUIREMENTS ARE 300 UNITS OF IRON, 200 UNITS OF VITAMIN A AND 240 UNITS OF CALCIUM, FIND THE NUMBER OF SERVINGS OF EACH TYPE OF FOOD WHICH WILL MEET THE NUTRITION REQUIREMENTS AT THE MINIMUM COST.

PROVIDE THE FOLLOWING INFORMATION:

1. THE INITIAL SIMPLEX MATRIX

PRE-INITIAL

$$\left[\begin{array}{ccc|c} 20 & 10 & 10 & 300 \\ 10 & 10 & 10 & 200 \\ 10 & 15 & 10 & 240 \\ \hline 20 & 24 & 18 & 0 \end{array}\right] \xrightarrow{\text{TRANS-POSE}} \left[\begin{array}{ccc|c} 20 & 10 & 10 & 20 \\ 10 & 10 & 15 & 24 \\ 10 & 10 & 10 & 18 \\ \hline 300 & 200 & 240 & 0 \end{array}\right] \Longrightarrow \left[\begin{array}{ccccccc|c} 20 & 10 & 10 & 1 & 0 & 0 & 0 & 20 \\ 10 & 10 & 15 & 0 & 1 & 0 & 0 & 24 \\ 10 & 10 & 10 & 0 & 0 & 1 & 0 & 18 \\ \hline -300 & -200 & -240 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

2. THE DETAILS OF ALL THE "mulrow" AND "pivot" COMMANDS YOU USE:

b2:=mulrow(b1,1,1/20);
b3:=pivot(b2,1,1);
b4:=mulrow(b3,2,1/10);
b5:=pivot(b4,2,3);
b6:=mulrow(b5,3,2/5);
b7:=pivot(b6,3,2);

3. MINIMUM FOR C: 428

FOOD 'L' = 10 FOOD 'M' = 8 FOOD 'N' = 2