Solutions EXAM #2WD
Math 2413 Fall 2018

7 pts ea

II

② $f(x) = 5x^5 + 3x^{-3} - 12x^{1/2}$

$\Rightarrow f'(x) = \boxed{25x^4 - 9x^{-4} - 6x^{-1/2}}$

③ $f(x) = \tan(3x) - \csc(-x)$

$\Rightarrow f'(x) = \boxed{\sec^2(3x)(3) - (-\csc(-x)\cot(-x))(-1)}$

$\text{OR} = \boxed{3\sec^2(3x) - \csc(-x)\cot(-x)}$

④ $f(x) = x^4 \sec(x)$

$\rightarrow f'(x) = \boxed{x^4 \sec(x)\tan(x) + \sec(x)(4x^3)}$

⑤ $f(x) = (-x^{-5} - x^{-2})^{-3}$

$\Rightarrow f'(x) = \boxed{(-3)(-x^{-5} - x^{-2})^{-4}(5x^{-6} + 2x^{-3})}$

⑥ $f(x) = \dfrac{x^{1/3} + \cos(x)}{4x}$

$\Rightarrow f'(x) = \boxed{\dfrac{(4x)(\frac{1}{3}x^{-2/3} - \sin(x)) - (x^{1/3} + \cos(x))(4)}{(4x)^2}}$

⑦ $f(x) = \sin^2(x^2)$

$\Rightarrow f'(x) = \boxed{2\sin(x^2)(\cos(x^2))(2x)} \overset{\text{or}}{=} \boxed{4x\sin(x^2)\cos(x^2)}$

8. $f(x) = (2-x^3)^{1/4}$

$\Rightarrow f'(x) = \boxed{\frac{1}{4}(2-x^3)^{-3/4}(-3x^2)}$

9. $f(x) = \dfrac{x^2+5}{x^5+2}$

$\Rightarrow f'(x) = \boxed{\dfrac{(x^5+2)(2x) - (x^2+5)(5x^4)}{(x^5+2)}}$

III

10. $f(x) = 2x^4 + 3x^{-2} - 12x$

$\Rightarrow f'(x) = 8x^3 - 6x^{-3} - 12$

$\Rightarrow f''(x) = \boxed{24x^2 + 18x^{-4}}$

11. $f(x) = x + \tan(2x)$

$\Rightarrow f'(x) = 1 + \sec^2(2x)(2)$

$= 1 + 2\sec^2(2x)$

$\Rightarrow f''(x) = 2(2)(\sec(2x))(\sec(2x)\tan(2x))(2)$

$= \boxed{8\sec^2(2x)\tan(2x)}$

IIII

(12) $x^3 + 5x^2y^2 = 3y$

$\Rightarrow 3x^2 + 5x^2(2y)\frac{dy}{dx} + y^2(10x) = 3\frac{dy}{dx}$

$\Rightarrow 10x^2y\frac{dy}{dx} - 3\frac{dy}{dx} = -3x^2 - 10xy^2$

$\Rightarrow \frac{dy}{dx} = \boxed{\frac{(-3x^2 - 10xy^2)}{(10x^2y - 3)}}$

(13) $3y - \cos(x+4y) = 2x$

$\Rightarrow 3\frac{dy}{dx} - (-\sin(x+4y))(1+4\frac{dy}{dx}) = 2$

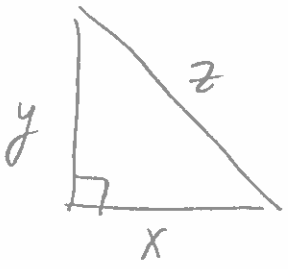
$\Rightarrow 3\frac{dy}{dx} + (\sin(x+4y))(1+4\frac{dy}{dx}) = 2$

$\Rightarrow 3\frac{dy}{dx} + \sin(x+4y) + 4\sin(x+4y)\frac{dy}{dx} = 2$

$\Rightarrow 3\frac{dy}{dx} + 4\sin(x+4y)\frac{dy}{dx} = 2 - \sin(x+4y)$

$\Rightarrow \frac{dy}{dx} = \boxed{\frac{2 - \sin(x+4y)}{3 + 4\sin(x+4y)}}$

V (14)



Find: $\frac{dy}{dt}$ = ? m/s

given: $\frac{dx}{dt} = 2$ m/s

$\frac{dz}{dt} = -1$ m/s

we have: $x^2+y^2=z^2$

$z=5$, $y=4$ $\Rightarrow x^2+y^2=z^2$

$\Rightarrow x^2+4^2=5^2$

$\Rightarrow x^2=9$

$\Rightarrow x=3$

$\Rightarrow 2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

$\Rightarrow (3)(2)+(4)\frac{dy}{dt}=(5)(-1)$

$\Rightarrow 6+4\frac{dy}{dt}=-5$

$\Rightarrow 4\frac{dy}{dt}=-11 \Rightarrow \frac{dy}{dt}=$ **$-11/4$ m/s** or **-2.75 m/s**

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(15) Find: $\frac{dA}{dt}$ = ? cm²/sec

given $\frac{dV}{dt}=72\pi$ cm³/s

$V=36\pi$

(A) we know $V=\frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt}=(3)(\frac{4}{3})\pi r^2\frac{dr}{dt}$

$=4\pi r^2\frac{dr}{dt}$

must find r:

$36\pi=\frac{4}{3}\pi r^3$

$\Rightarrow \frac{3}{4\pi}(36\pi)=r^3$

$\Rightarrow 27=r^3 \Rightarrow r=3$

so $72\pi=4\pi(3)^2\frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt}=\frac{72\pi}{9\cdot 4\cdot\pi}$

$=\frac{72\pi}{36\pi}$

$=2$

so $A=4\pi r^2$

$\Rightarrow \frac{dA}{dt}=4\pi 2r\frac{dr}{dt}$

$\Rightarrow =8\pi(3)(2)$

$=$ **48π cm²/sec**