Solutions

① $\lim_{x\to(-3)} \frac{x^2+4x+3}{x^2-9} = \lim_{x\to(-3)} \frac{(x+3)(x+1)}{(x+3)(x-3)} = \lim_{x\to(-3)} \frac{(x+1)}{(x-3)} = \frac{-3+1}{-3-3} = \frac{-2}{-6} = \boxed{\frac{1}{3}}$

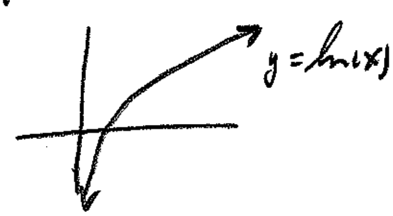
② $\lim_{x\to\infty} \frac{x}{-3x+5} = \lim_{x\to\infty} \frac{x}{-3x+5}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{x}{x}}{\frac{-3x}{x}+\frac{5}{x}} = \lim_{x\to\infty} \frac{1}{-3+\cancel{\frac{5}{x}}^{\to 0}} = \boxed{-\frac{1}{3}}$

③ $\lim_{x\to\infty} \frac{x^2-2x}{2x^2+3x+5} = \lim_{x\to\infty} \frac{x^2-2x}{2x^2+3x+5}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\frac{x^2}{x^2}-\cancel{\frac{2x}{x^2}}^{\to 0}}{\frac{2x^2}{x^2}+\cancel{\frac{3x}{x^2}+\frac{5}{x^2}}^{\to 0}}$

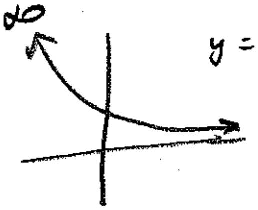
$= \lim_{x\to\infty} \frac{1}{2} = \boxed{\frac{1}{2}}$

④ $\lim_{x\to 4} \frac{-5}{(x-4)^2} = \frac{-5}{0^+} = \boxed{-\infty}$ ⑤ $\lim_{x\to 1^+} \frac{x+3}{x-1} = \frac{4}{0^+} = \boxed{\infty}$

⑥ $\lim_{x\to\infty} \ln(x) = \boxed{\infty}$



⑦ $\lim_{x\to\infty} e^{-x} = \boxed{0}$



⑧ $f(x) = \frac{\cos(3x)}{x-\sin(3x)} \Rightarrow f'(x) = \frac{(x-\sin(3x))(-\sin(3x)(3)) - (\cos(3x))(1-\cos(3x)(3))}{(x-\sin(3x))^2}$

$= \frac{-3x\sin(3x)+3\sin^2(3x)-\cos(3x)+3\cos^2(3x)}{(x-\sin(3x))^2}$

$= \frac{-3x\sin(3x)-\cos(3x)+3[\sin^2(3x)+\cos^2(3x)]}{(x-\sin(3x))^2}$

$= \boxed{\frac{-3x\sin(3x)-\cos(3x)+3}{(x-\sin(3x))^2}}$

9. $y = e^{4x}\sqrt{x^4-9} = e^{4x}(x^4-9)^{1/2}$

$y' = e^{4x}\left(\frac{1}{2}\right)(x^4-9)^{-1/2}(4x^3) + (x^4-9)^{1/2}(e^{4x})(4)$

$= 2x^3e^{4x}(x^4-9)^{-1/2} + 4e^{4x}(x^4-9)^{1/2}$

$= 2e^{4x}(x^4-9)^{-1/2}\left[x^3+2(x^4-9)\right]$

$= \boxed{\dfrac{2e^{4x}\left[2x^4+x^3-18\right]}{(x^4-9)^{1/2}}}$

10. $x^2y - 3\tan(y) = 4$ use implicit differentiation:

$\Rightarrow x^2\frac{dy}{dx} + y(2x) - 3\sec^2(y)\frac{dy}{dx} = 0$

$\Rightarrow x^2\frac{dy}{dx} - 3\sec^2(y)\frac{dy}{dx} = -2xy$

$\Rightarrow \frac{dy}{dx}\left[x^2-3\sec^2(y)\right] = -2xy \Rightarrow \frac{dy}{dx} = \boxed{\dfrac{-2xy}{x^2-3\sec^2(y)}}$

11. $y = ex - 9\pi^2 + \ln(3)$ Note that $9\pi^2$ and $\ln(3)$ are simply constants.

$\Rightarrow \boxed{\frac{dy}{dx} = e}$

Also note that ex is NOT e^x!
It is regarded like $4x$.

(12). $2e^x e^y - \cos^3(x) + \sin^3(y) = 1$ Implicit differentiation

$$\Rightarrow 2e^x \cdot e^y \frac{dy}{dx} + e^y(2e^x) - 3\cos^2(x)\cdot(-\sin(x)) + 3\sin^2(y)\left(\cos(y)\frac{dy}{dx}\right) = 0$$

$$\Rightarrow 2e^x e^y \frac{dy}{dx} + 2e^x e^y + 3\sin(x)\cos^2(x) + 3\sin^2(y)\cos(y)\frac{dy}{dx} = 0$$

$$\Rightarrow 2e^x e^y \frac{dy}{dx} + 3\sin^2(y)\cos(y)\frac{dy}{dx} = -2e^x e^y - 3\sin(x)\cos^2(x)$$

$$\Rightarrow \frac{dy}{dx} = \boxed{\frac{-2e^x e^y - 3\sin(x)\cos^2(x)}{2e^x e^y + 3\sin^2(y)\cos(y)}}$$

(13) $y = \sqrt[3]{\sin(e^{5x})} = \left(\sin(e^{5x})\right)^{1/3}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{3}\left(\sin(e^{5x})\right)^{-2/3}\left(\cos(e^{5x})\left(e^{5x}(5)\right)\right)$$

$$= \boxed{\frac{5e^{5x}\cos(e^{5x})}{3\left(\sin(e^{5x})\right)^{2/3}}}$$

(14). $\int \left(3x^4 + \sqrt{x} - \frac{2}{x^4}\right)dx = \int \left(3x^4 + x^{1/2} - 2x^{-4}\right)dx$

$$= \frac{3x^5}{5} + \frac{x^{3/2}}{3/2} - \frac{2x^{-3}}{-3} + C = \boxed{\frac{3x^5}{5} + \frac{2x^{3/2}}{3} + \frac{2x^{-3}}{3} + C}$$

(15) $\int \sin(3x)\,dx = \boxed{-\frac{1}{3}\cos(3x) + C}$ BASIC FORM

(16) $\int \sec^2(x)\,dx = \tan(x) + C$ (Basic form)

(17) $\int 2x\sqrt{x^2-9}\,dx = \int 2x\,(x^2-9)^{1/2}\,dx$

Let $u = x^2-4$ $\quad = \int u^{1/2}\,du$

$du = 2x\,dx$ $\quad = \frac{2u^{3/2}}{3} + C = \boxed{\frac{2(x^2-9)^{3/2}}{3} + C}$

(18). $\int x^3 (x^4-16)^3\,dx \quad = \frac{1}{4}\int u^3\,du$

Let $u = x^4-16$ $\quad = \frac{1}{4}\,\frac{u^4}{4} + C$

$du = 4x^3\,dx$

$\Rightarrow \frac{1}{4}du = x^3 dx$ $\quad = \boxed{\frac{1}{16}(x^4-16)^4 + C}$

(19). $\int \sin^5(x)\cos(x)\,dx \quad = \int u^5\,du$

Let $u = \sin(x)$ $\quad = \frac{u^6}{6} + C = \boxed{\frac{\sin^6(x)}{6} + C}$

$du = \cos(x)\,dx$

(20). $\int 3x\cos(4x^2)\,dx \quad = \frac{3}{8}\int \cos(u)\,du$

Let $u = 4x^2$

$\Rightarrow du = 8x\,dx$ $\quad = -\frac{3}{8}\sin(u) + C$

$\Rightarrow \frac{1}{8}du = x\,dx$ $\quad = \boxed{-\frac{3}{8}\sin(4x^2) + C}$

$\Rightarrow \frac{3}{8}du = 3x\,dx$

(21). $\int \frac{3x^2}{x^3-1}\,dx \quad = \int \frac{1}{u}\,du$

Let $u = x^3-1$ $\quad = \ln|u| + C$

$\Rightarrow du = 3x^2$ $\quad = \boxed{\ln|x^3-1| + C}$

(22). $\int \frac{dx}{(2x-3)^4} = \frac{1}{2}\int \frac{1}{u^4}\,du$

Let $u = 2x-3$

$du = 2\,dx$

$\Rightarrow \frac{1}{2}du = dx$

$= \frac{1}{2}\int u^{-4}\,du$

$= \frac{1}{2}\frac{u^{-3}}{-3} + C$

$= \boxed{-\frac{1}{6}(2x-3)^{-3} + C}$

(23). $\int \frac{\ln(x)}{x}\,dx = \int u\,du$

Let $u = \ln(x)$

$du = \frac{1}{x}\,dx$

$= \frac{u^2}{2} + C$

$= \boxed{\frac{(\ln(x))^2}{2} + C}$

(24). $\int \frac{x^4-5}{x^2}\,dx = \int\left[\frac{x^4}{x^2} - \frac{5}{x^2}\right]dx$

$= \int\left[x^2 - 5x^{-2}\right]dx$

$= \frac{x^3}{3} - \frac{5x^{-1}}{-1} + C = \boxed{\frac{1}{3}x^3 + 5x^{-1} + C}$

(25). $\int \frac{e^{2x}}{e^{2x}-1}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \boxed{\frac{1}{2}\ln|e^{2x}-1| + C}$

Let $u = e^{2x}-1$

$\Rightarrow du = e^{2x}(2)\,dx$

$\Rightarrow \frac{1}{2}du = e^{2x}\,dx$