# CALC I EXAM I



## SPRING 2019

(I) ______

(II) (2) $\lim\limits_{x \to \pi} \dfrac{\cos(x)+1}{1+\sin(x)} = \dfrac{\cos(\pi)+1}{1+\sin(\pi)} = \dfrac{-1+1}{1+0} = \dfrac{0}{1} = \boxed{0}$

(3) $\lim\limits_{x \to 4} \dfrac{x^2-6x+8}{x^2-x-12} = \lim\limits_{x \to 4} \dfrac{\cancel{(x-4)}(x-2)}{\cancel{(x-4)}(x+3)} = \lim\limits_{x \to 4} \dfrac{x-2}{x+3} = \dfrac{4-2}{4+3} = \boxed{\dfrac{2}{7}}$

(4) $\lim\limits_{h \to 0} \dfrac{(a+h)^2-3(a+h)+2-(a^2-3a+2)}{h} =$

$= \lim\limits_{h \to 0} \dfrac{\cancel{a^2}+2ah+h^2-\cancel{3a}-3h+\cancel{2}-\cancel{a^2}+\cancel{3a}-\cancel{2}}{h}$

$= \lim\limits_{h \to 0} \dfrac{2ah+h^2-3h}{h}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{h}(2a+h-3)}{\cancel{h}}$

$= \lim\limits_{h \to 0} (2a+h-3)$

$= 2a+0-3$

$= \boxed{2a-3}$

(5) $\lim\limits_{x \to (-1)} (x^3-4x^2-6x) = (-1)^3-4(-1)^2-6(-1)$

$= -1-4+6$

$= \boxed{1}$

(6) $$\lim_{x\to 0} \frac{\tan(x)}{x\cos(x)} = \lim_{x\to 0} \frac{\sin(x)}{x \cdot \cos(x)\cos(x)}$$

$$= \lim_{x\to 0} \frac{\sin(x)}{x} \cdot \frac{1}{\cos^2(x)}$$

$$= 1 \cdot \frac{1}{\cos^2(0)}$$

$$= \boxed{1}$$

(7) $$\lim_{x\to 0} \frac{5}{x\cot(x)} = \lim_{x\to 0} \frac{5}{x \cdot \frac{\cos(x)}{\sin(x)}}$$

$$= \lim_{x\to 0} 5 \cdot \frac{\sin(x)}{x \cdot \cos(x)}$$

$$= \lim_{x\to 0} 5 \cdot \frac{\sin(x)}{x} \cdot \frac{1}{\cos(x)}$$

$$= 5 \cdot 1 \cdot 1$$

$$= \boxed{5}$$

(8) $\lim_{x\to(-1)} f(x)$ where $f(x) = \begin{cases} 4x - x^2 & \text{if } x \le -1 \\ \frac{15}{x+4} & \text{if } x > -1 \end{cases}$

(i) $$\lim_{x\to(-1)^-} f(x) \underset{=}{\text{TOP}} \lim_{x\to(-1)^-} (4x - x^2) = 4(-1) - (-1)^2 = -4 - 1 = -5$$

(ii) $$\lim_{x\to(-1)^+} f(x) \underset{=}{\text{BOT}} \lim_{x\to(-1)^+} \left(\frac{15}{x+4}\right) = \frac{15}{(-1)+4} = \frac{15}{3} = 5$$

(iii) These do not equal so

$$\lim_{x\to(-1)} f(x) \text{ D.N.E.}$$

(9) $\lim_{x\to 4} \frac{(\sqrt{x+21}-5)\cdot(\sqrt{x+21}+5)}{(x-4)\cdot(\sqrt{x+21}+5)} =$

$= \lim_{x\to 4} \frac{x+21-25}{(x-4)(\sqrt{x+21}+5)}$

$= \lim_{x\to 4} \frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x+21}+5}$

$= \lim_{x\to 4} \frac{1}{\sqrt{x+21}+5} = \frac{1}{\sqrt{25}+5} = \frac{1}{5+5} = \boxed{\frac{1}{10}}$

(10) $\lim_{x\to 5} f(x)$ where $f(x) = \begin{cases} x^2-4 & \text{if } x \le 5 \\ 4x+2 & \text{if } x > 5 \end{cases}$

(i) $\lim_{x\to 5^-} f(x) \overset{\text{TOP}}{=} \lim_{x\to 5^-} (x^2-4) = (5)^2-4 = 21$

(ii) $\lim_{x\to 5^+} f(x) \overset{\text{BOT}}{=} \lim_{x\to 5^+} (4x+2) = 4(5)+2 = 22$

(iii) since These do NOT Equal

$\lim_{x\to 5} f(x)$ $\boxed{\text{D.N.E.}}$

(11). $\lim_{x\to 2} \frac{x-6}{x-2} \to \frac{-4}{0}$

if $x = 1.9 \to \frac{\text{Neg}}{\text{Neg}} \to +\infty$

if $x = 2.1 \to \frac{\text{Neg}}{\text{pos}} \to -\infty$

$\Bigg\} \Rightarrow$ SINCE THESE DO NOT AGREE,

$\lim_{x\to 2} \frac{x-6}{x-2}$ $\boxed{\text{D.N.E.}}$

III

(12) $f(x) = \dfrac{x-6}{x^3-4x^2}$

$f(x)$ is NOT continuous when

$x^3-4x^2=0$

$\Rightarrow x^2(x-4)=0$

$\Rightarrow x=0 \quad x=4$

$\boxed{\text{NOT CONTINUOUS at } x=0 \quad x=4}$

(13) $f(x) = \begin{cases} -\sin(x) & \text{at } x \le 0 \\ \tan(x) & \text{at } x > 0 \end{cases}$ on $(-2\pi, 2\pi)$

must check $x=0$

(i) $f(0) \overset{\text{TOP}}{=} -\sin(0) = 0$

(ii) $\lim_{x\to 0} f(x)$

$\lim_{x\to 0^-} f(x) \overset{\text{TOP}}{=} \lim_{x\to 0^-}(-\sin(x)) = -\sin(0) = 0$

$\lim_{x\to 0^+} f(x) \overset{\text{BOT}}{=} \lim_{x\to 0} \tan(x) = \tan(0) = 0$

$\therefore \lim_{x\to 0} f(x) = 0$

(iii) These agree $\Rightarrow$ continuous at $x=0$

$\boxed{\text{ONLY DISCONTINUITIES } x = \pi/2 \; ; \; x = 3\pi/2}$

V 17 Prove $\lim_{x\to(-3)} (-5x-6) = 9$

(I) let $\epsilon > 0$

(II) Search for $\delta$:

we want
$|f(x)-L| < \epsilon \Rightarrow |(-5x-6)-9| < \epsilon$

$\Rightarrow |-5x-15| < \epsilon$

$\Rightarrow |-5(x+3)| < \epsilon$

$\Rightarrow |-5|\,|x-(-3)| < \epsilon$

$\Rightarrow |x-(-3)| < \frac{\epsilon}{|-5|}$

Choose $\delta = \frac{\epsilon}{|-5|}$

(III) Verify $\delta$:

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-(-3)| < \frac{\epsilon}{|-5|}$

$\Rightarrow |-5|\,|x+3| < \epsilon$

$\Rightarrow |-5(x+3)| < \epsilon$

$\Rightarrow |-5x-15| < \epsilon$

$\Rightarrow |-5x-6+6-15| < \epsilon$

$\Rightarrow |(-5x-6)-9| < \epsilon$

$\Rightarrow |f(x)-L| < \epsilon$

---

IIII 16

$$f(x) = \begin{cases} x^2 - kx & \text{if } x < -2 \\ 4x+2 & \text{if } x \geq -2 \end{cases}$$

we require

$$\lim_{x\to(-2)^-} f(x) = \lim_{x\to(-2)^+} f(x)$$

$$\Rightarrow \lim_{x\to(-2)} (x^2 kx) = \lim_{x\to(-2)^+} (4x+2)$$

$$\Rightarrow (-2)^2 - k(-2) = 4(-2)+2$$

$$\Rightarrow 4+2k = -6$$

$$\Rightarrow 2k = -10$$

$$\Rightarrow \boxed{k = -5}$$