SOLUTIONS CALC II

PG Ire

EXAM I SPRING 2019

II

① $\int \cos^2(x)\,dx = \frac{1}{2}\int [1+\cos(2x)]\,dx$

$= \boxed{\frac{1}{2}\left[x+\frac{1}{2}\sin(2x)\right]+C}$

② $\int 8x^2\sin(2x)\,dx$

parts: $u=8x^2$ $\quad v=-\frac{1}{2}\cos(2x)$

$du=16x\,dx$ $\quad dv=\sin(2x)\,dx$

$= \left(-\frac{1}{2}\right)(8)x^2\cos(2x) - \int\left(-\frac{1}{2}\right)(16)x\cos(2x)\,dx$

$= -4x^2\cos(2x)+\int 8x\cos(2x)\,dx$

parts again

$u=8x$ $\quad v=\frac{1}{2}\sin(2x)$

$du=8\,dx$ $\quad dv=\cos(2x)\,dx$

$= -4x^2\cos(2x)+\left[\left(\frac{1}{2}\right)(8x)\sin(2x)-\int(8)\left(\frac{1}{2}\right)\sin(2x)\,dx\right]$

$= -4x^2\cos(2x)+4x\sin(2x)-\int 4\sin(2x)\,dx$

$= -4x^2\cos(2x)+4x\sin(2x)-4\left(-\frac{1}{2}\right)\cos(2x)+C$

$= \boxed{-4x^2\cos(2x)+4x\sin(2x)+2\cos(2x)+C}$

(3) $\int \sin^3(x)\cos^2(x)\,dx =$

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$= \int \sin^2(x)\cos^2(x)\cdot \underline{\sin(x)\,dx}$

$= \int [1-\cos^2(x)]\cdot\cos^2(x)\cdot \underline{\sin(x)\,dx}$

$= \int [\cos^2(x)-\cos^4(x)]\cdot \underline{\sin(x)\,dx}$

Let $u=\cos(x)$

$\Rightarrow du=-\sin(x)\,dx$

$\Rightarrow -du=\sin(x)\,dx$

$= -\int [u^2-u^4]\,du$

$= -\left[\frac{u^3}{3}-\frac{u^5}{5}\right]+C$

$= \boxed{\frac{-\cos^3(x)}{3}+\frac{\cos^5(x)}{5}+C}$

(4) $\int \tan^3(x)\sec^4(x)\,dx =$

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OPTION I

$= \int \tan^3(x)\cdot\sec^2(x)\cdot\underline{\sec^2(x)\,dx}$

$= \int \tan^3(x)[\tan^2+1]\cdot\underline{\sec^2(x)\,dx}$

$= \int [\tan^5(x)+\tan^3(x)]\cdot\underline{\sec^2(x)\,dx}$

Let $u=\tan(x)$

$du=\sec^2(x)\,dx$

$= \int [u^5+u^3]\,du$

$= \frac{u^6}{6}+\frac{u^4}{4}+C$

$= \boxed{\frac{\tan^6(x)}{6}+\frac{\tan^4(x)}{4}+C}$

OR OPTION II

$= \int \tan^2(x)\sec^3(x)\cdot\underline{\sec(x)\tan(x)\,dx}$

$= \int [\sec^2(x)-1]\cdot\sec^3(x)\cdot\underline{\sec(x)\tan(x)\,dx}$

$= \int [\sec^5(x)-\sec^3(x)]\cdot\underline{\sec(x)\tan(x)\,dx}$

Let $u=\sec(x)$

$du=\sec(x)\tan(x)\,dx$

$= \int [u^5-u^3]\cdot du$

$= \frac{u^6}{6}-\frac{u^4}{4}+C$

$= \boxed{\frac{\sec^6(x)}{6}-\frac{\sec^4(x)}{4}+C}$

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(5) $\displaystyle\int \frac{x^3}{16\sqrt{16-x^2}}\,dx$

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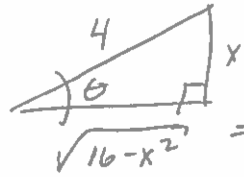

choose $\sin(\theta) = \frac{x}{4}$

$\Rightarrow 4\sin(\theta) = x$

$\Rightarrow 4\cos(\theta)\,d\theta = dx$

choose $\cos(\theta) = \frac{\sqrt{16-x^2}}{4}$

$\Rightarrow 4\cos(\theta) = \sqrt{16-x^2}$

$$= \int \frac{(4\sin(\theta))^3 \cdot \cancel{4\cos(\theta)}}{16 \cdot \cancel{4\cos(\theta)}}\,d\theta$$

$$= \frac{64}{16}\int \sin^3(\theta)\,d\theta$$

$$= 4\int \sin^2(\theta)\cdot\sin(\theta)\,d\theta$$

$$= 4\int [1-\cos^2(\theta)]\cdot\sin(\theta)\,d\theta$$

Let $u = \cos(\theta)$

$du = -\sin(\theta)\,d\theta$

$\Rightarrow -du = \sin(\theta)\,d\theta$

$$= -4\int [1-u^2]\,du$$

$$= -4\left[u - \frac{u^3}{3}\right] + C$$

$$= -4\left[\cos(\theta) - \frac{\cos^3(\theta)}{3}\right] + C$$

$$= \boxed{-4\left[\frac{\sqrt{16-x^2}}{4} - \frac{1}{3}\left(\frac{\sqrt{16-x^2}}{4}\right)^3\right] + C}$$

(6) $\displaystyle\int \frac{x^2}{\sqrt{x^2+25}}\,dx$

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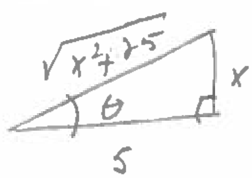

choose $\tan(\theta) = \frac{x}{5}$

$\Rightarrow 5\tan(\theta) = x$

$\Rightarrow 5\sec^2(\theta)\,d\theta = dx$

choose $\sec(\theta) = \frac{\sqrt{x^2+25}}{5}$

$\Rightarrow 5\sec(\theta) = \sqrt{x^2+25}$

$$= \int \frac{(5\tan(\theta))^2 \cdot \cancel{5}\sec^{\cancel{2}}(\theta)}{\cancel{5\sec(\theta)}}\,d\theta$$

$$= 25\int \tan^2(\theta)\cdot\sec(\theta)\,d\theta$$

$$= 25\int [\sec^2(\theta)-1]\cdot\sec(\theta)\,d\theta$$

$$= 25\int [\sec^3(\theta) - \sec(\theta)]\,d\theta$$

$$= 25\left[\frac{1}{2}\left(\ln|\sec(\theta)+\tan(\theta)| + \sec(\theta)\tan(\theta)\right) - \ln|\sec(\theta)+\tan(\theta)|\right] + C$$

$$= \boxed{25\left[\frac{1}{2}\left(\ln\left|\frac{\sqrt{x^2+25}}{5} + \frac{x}{5}\right| + \frac{\sqrt{x^2+25}}{5}\cdot\frac{x}{5}\right) - \ln\left|\frac{\sqrt{x^2+25}}{5} + \frac{x}{5}\right|\right] + C}$$

#7. $\displaystyle\int \frac{x^2-x+2}{(x+1)(x-1)^2}\,dx =$

<u>par frac</u>: $\displaystyle\frac{x^2-x+2}{(x+1)(x-1)^2} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$

$\Rightarrow x^2-x+2 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$

Let $x=1$

$\Rightarrow 2 = 0+0+C(2) \Rightarrow C=1$

Let $x=-1$

$\Rightarrow 4 = A(4)+0+0 \Rightarrow A=1$

Let $x=0$

$\Rightarrow 2 = (1)(1)+B(-1)+(1)(1) \Rightarrow B=0$

$\displaystyle = \int \frac{1}{x+1}\,dx + \int \frac{1}{(x-1)^2}\,dx$

$\displaystyle = \int \frac{1}{x+1}\,dx + \int (x-1)^{-2}\,dx \longrightarrow = \boxed{\ln|x+1| + \frac{(x-1)^{-1}}{-1} + C}$

#8 $\displaystyle\int \frac{7x^2+12}{x^3+4x}\,dx$

<u>par fract</u>

$\displaystyle\frac{7x^2+12}{x(x^2+4)} = \frac{A}{x} + \frac{(Bx+C)}{(x^2+4)}$

$\Rightarrow 7x^2+12 = A(x^2+4) + (Bx+C)(x)$

$= Ax^2+4A+Bx^2+Cx$

$= (A+B)x^2+Cx+4A$

$\Rightarrow A+B = 7$; $3+B=7 \Rightarrow B=4$

$C = 0 \Rightarrow C=0$

$4A = 12 \Rightarrow A=3$

so we have:

$\displaystyle = \int \frac{3}{x}\,dx + \int \frac{4x}{x^2+4}\,dx$

$\displaystyle \int \frac{4x}{x^2+4}\,dx$

Let $u = x^2+4$

$\Rightarrow du = 2x\,dx$

$\Rightarrow 2du = 4x\,dx$

$\displaystyle = 2\int \frac{1}{u}\,du \Rightarrow 2\ln|x^2+4|$

$= \boxed{3\ln|x| + 2\ln|x^2+4| + C}$

(9) $\int \frac{2x-a-b}{(x-a)(x-b)}\,dx$

par frac:

$\frac{2x-a-b}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$

$\Rightarrow 2x-a-b = A(x-b) + B(x-a)$

Let $x=a$

$\Rightarrow a-b = A(a-b) + 0 \Rightarrow A = 1$

Let $x=b$

$\Rightarrow -a+b = 0 + B(b-a) \Rightarrow B = 1$

So we have

$= \int \frac{1}{x-a}\,dx + \int \frac{1}{x-b}\,dx$

$\boxed{= \ln|x-a| + \ln|x-b| + C}$

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(10) $\lim_{x\to 0} \frac{\cos(x)-1}{e^x-1} \overset{L'H}{=} \lim_{x\to 0} \frac{-\sin(x)}{e^x}$

$= \frac{-\sin(0)}{e^0}$

$= \frac{0}{1}$

$= \boxed{0}$

(11) $\lim_{x\to\infty} (e^x+1)^{1/x} \to \infty^0$

Let $y = \lim_{x\to\infty} (e^x+1)^{1/x}$

$\Rightarrow \ln(y) = \ln\left[\lim_{x\to\infty} (e^x+1)^{1/x}\right]$

$= \lim_{x\to\infty} \left[\ln (e^x+1)^{\frac{1}{x}}\right]$

$= \lim_{x\to\infty} \left[\frac{1}{x} \cdot \ln(e^x+1)\right]$

$= \lim_{x\to\infty} \left[\frac{\ln(e^x+1)}{x}\right] \to \frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty} \left[\frac{\frac{1}{e^x+1}(e^x)}{1}\right]$

$= \lim_{x\to\infty} \left[\frac{e^x}{e^x+1}\right] \to \frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty} \left[\frac{e^x}{e^x}\right]$

$= \lim_{x\to\infty} (1)$

$= 1$

So

$\ln(y) = 1$

$\Rightarrow e^{\ln(y)} = e^1$

$\Rightarrow \boxed{y = e}$.

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$$\int_2^\infty \frac{6}{(x-1)^4}\,dx = \lim_{b\to\infty}\left[6\int_2^b \frac{1}{(x-1)^4}\,dx\right]$$

$$= \lim_{b\to\infty}\left[6\int_2^b (x-1)^{-4}\,dx\right]$$

$$= \lim_{b\to\infty}\left[6\left[\frac{(x-1)^{-3}}{-3}\right]_2^b\right]$$

$$= \lim_{b\to\infty}\left[-2\left[\left((b-1)^{-3}\right) - \left((2-1)^{-3}\right)\right]\right]$$

$$= -2\cdot\lim_{b\to\infty}\left[\frac{1}{(b-1)^3} - 1\right]$$

$$= -2\left[\frac{1}{\infty} - 1\right]$$

$$= -2[0-1]$$

$$= -2[-1]$$

$$= \boxed{2}$$