

I. FIND EACH LIMIT. IF A LIMIT FAILS TO EXIST, STATE THAT FACT.  
IF A LIMIT VALUE IS  $\infty$  OR  $-\infty$ , STATE THAT FACT.

1.  $\lim_{x \rightarrow 2\pi} \frac{5 - \cos(x)}{\sin(x) + 2}$

2.  $\lim_{x \rightarrow (-3)} \frac{x^2 + 8x + 15}{x^2 - x - 12}$

3.  $\lim_{h \rightarrow 0} \frac{2(a+h)^2 + 4(a+h) - 5 - (2a^2 + 4a - 5)}{h}$

4.  $\lim_{x \rightarrow 2} (3x^3 - 5x^2 - 6|x-1|)$

5.  $\lim_{x \rightarrow 0} \frac{\sin(8x)\tan(x)}{2x^2}$

6.  $\lim_{x \rightarrow 0} \frac{\tan(x)}{x \sec(x)}$

Hint: Let  $2x^2 = 2x \cdot x$  and split into two fractions

7.  $\lim_{x \rightarrow -5} \frac{\sqrt{x+14} - 3}{x+5}$

8.  $\lim_{x \rightarrow 4^+} \frac{x+8}{x-4}$

9.  $\lim_{x \rightarrow 2} f(x)$  where  $f(x) = \begin{cases} 4x - x^2, & x < 2 \\ \frac{18}{x+4}, & x \geq 2 \end{cases}$

10.  $\lim_{x \rightarrow 5} f(x)$  where  $f(x) = \begin{cases} x^2 - 1, & x \leq 2 \\ 3x - 4, & x > 2 \end{cases}$

11.  $\lim_{x \rightarrow \pi} f(x)$  where  $f(x) = \begin{cases} \tan(x) - 3\cos(x), & x < \pi \\ \frac{6}{\sin(x)+2}, & x \geq \pi \end{cases}$

12.  $\lim_{x \rightarrow 3^-} \frac{-5x}{3-x}$

II. FIND THE  $x$ -VALUES WHERE EACH FUNCTION IS NOT CONTINUOUS.

13.  $f(x) = \frac{x^3}{x^3 - 9x}$

14.  $f(x) = \sec(x)$

$$15. \quad f(x) = \begin{cases} \frac{8x}{x^2 + 1}, & x \leq -1 \\ \frac{4}{x}, & x > -1 \end{cases}$$

$$16. \quad f(x) = \begin{cases} -\cos(x), & x \leq \pi \\ x, & x > \pi \end{cases}$$

III. FIND THE VALUE OF  $k$  WHICH WILL MAKE THE FUNCTION CONTINUOUS.

$$17. \quad f(x) = \begin{cases} x^2 + 4x + 2 & x < 1 \\ kx + 3 & x \geq 1 \end{cases}$$

$$18. \quad f(x) = \begin{cases} x^2 - \sin(x) + k_1 & x < \pi \\ x^2 - \cos(x) - 2 & \pi \leq x < 2\pi \\ x^2 - \tan(x) + k_2 & x \geq 2\pi \end{cases}$$

III. USING A  $\delta - \varepsilon$  PROOF, PROVE THE FOLLOWING STATEMENTS.

$$19. \quad \lim_{x \rightarrow 2} (4x - 1) = 7$$

$$20. \quad \lim_{x \rightarrow (-2)} (-3x + 4) = 10$$