SOLUTIONS Practice Exam # 1ne
CALC II FALL 2017

I

① $\int \sin^2(x)\,dx = \frac{1}{2}\int [1-\cos(2x)]\,dx$

$= \boxed{\frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right] + C}$

② $\int \sec^5(x)\,dx = \int \sec^3(x)\cdot\sec^2(x)\,dx$

parts:

$u = \sec^3(x)$ $\quad\quad v = \tan(x)$

$du = 3\sec^2(x)\cdot\sec(x)\tan(x)\,dx$ $\quad\quad dv = \sec^2(x)\,dx$

$= \sec^3(x)\tan(x) - \int 3\sec^3(x)\tan^2(x)\,dx$

$= \sec^3(x)\tan(x) - 3\int \sec^3(x)\cdot[\sec^2(x)-1]\,dx$

$= \sec^3(x)\tan(x) - 3\int \sec^5(x)\,dx + 3\int \sec^3(x)\,dx$

$= \sec^3(x)\tan(x) - 3\int \sec^5(x)\,dx + 3\left(\frac{1}{2}\right)\left[\sec(x)\tan(x) + \ln|\sec(x)+\tan(x)|\right]$

move to left-hand side

so

$4\int \sec^5(x)\,dx = \sec^3(x)\tan(x) + \frac{3}{2}\sec(x)\tan(x) + \frac{3}{2}\ln|\sec(x)+\tan(x)| + C$

$\Rightarrow \int \sec^5(x)\,dx = \boxed{\frac{1}{4}\sec^3(x)\tan(x) + \frac{3}{8}\sec(x)\tan(x) + \frac{3}{8}\ln|\sec(x)+\tan(x)| + C}$

③ $\int 4x^2 e^{(2x)}\,dx =$

parts $u = 4x^2$ $\quad v = \frac{1}{2}e^{(2x)}$

$du = 8x\,dx$ $\quad dv = e^{(2x)}\,dx$

$= 4x^2\left(\frac{1}{2}\right)e^{(2x)} - (8)\left(\frac{1}{2}\right)\int x e^{(2x)}\,dx$

$= 2x^2 e^{(2x)} - 4\int x e^{(2x)}\,dx$

Parts: $u = x$ $\quad v = \frac{1}{2}e^{(2x)}$

$du = dx$ $\quad dv = e^{(2x)}\,dx$

$= 2x^2 e^{(2x)} - \left[4\left(\frac{1}{2}\right)x e^{(2x)} - 4\left(\frac{1}{2}\right)\int e^{(2x)}\,dx\right]$

$= 2x^2 e^{(2x)} - 2x e^{(2x)} - 2\left(\frac{1}{2}\right)e^{(2x)} + C$

$= \boxed{2x^2 e^{(2x)} - 2x e^{(2x)} - e^{(2x)} + C}$

④ $\int \cos^3(x)\sin^4(x)\,dx = \int \cos^2(x)\sin^4(x)\,\underbrace{\cos(x)\,dx}_{\text{reserved}}$

$= \int [1-\sin^2(x)]\sin^4(x)\,\cos(x)\,dx$

$= \int [\sin^4(x) - \sin^6(x)]\,\cos(x)\,dx$

Let $u = \sin(x)$

$du = \cos(x)\,dx$

$= \int [u^4 - u^6]\,du$

$= \frac{u^5}{5} - \frac{u^7}{7}$

$= \boxed{\frac{\sin^5(x)}{5} - \frac{\sin^7(x)}{5} + C}$

(5). $\int \tan^3(x)\sec(x)\,dx = \int \tan^2(x)\cdot \underline{\sec(x)\tan(x)\,dx}$

$= \int [\sec^2(x) - 1]\,\underline{\sec(x)\tan(x)\,dx}$

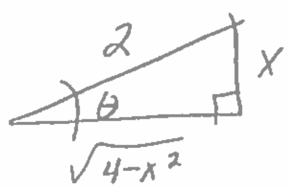
Let $u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

$= \int [u^2 - 1]\,du$

$= \frac{u^3}{3} - u + C = \boxed{\frac{\sec^3(x)}{3} - \sec(x) + C}$

(6). $\int \frac{x}{2\sqrt{4-x^2}}\,dx =$



choose $\sin(\theta) = \frac{x}{2}$

$\Rightarrow 2\sin(\theta) = x$

$\Rightarrow 2\cos(\theta)\,d\theta = dx$

choose $\cos(\theta) = \frac{\sqrt{4-x^2}}{2}$

$\Rightarrow 2\cos(\theta) = \sqrt{4-x^2}$

$= \int \frac{\cancel{2}\sin(\theta)\cdot\cancel{2\cos(\theta)}}{\cancel{2}\cdot\cancel{2\cos(\theta)}}\,d\theta$

$= \int \sin(\theta)\,d\theta$

$= -\cos(\theta) + C$

$= \boxed{-\frac{\sqrt{4-x^2}}{2} + C}$

OR $\int \frac{x}{2\sqrt{4-x^2}}\,dx =$

Let $u = 4 - x^2$

$du = -2x\,dx$

$\Rightarrow -\frac{1}{2}du = x\,dx$

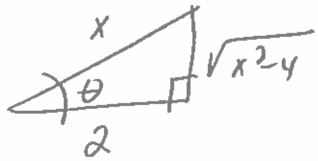
$= -\frac{1}{2}\int \frac{1}{2\sqrt{u}}\,du$

$= -\frac{1}{4}\int u^{-1/2}\,du$

$= -\frac{1}{4}\frac{u^{1/2}}{\frac{1}{2}} + C$

$= \boxed{-\frac{1}{2}\sqrt{4-x^2} + C}$

⑦ $$\int \frac{x^3}{\sqrt{x^2-4}}\,dx =$$



choose
$\sec(\theta) = \frac{x}{2}$
$\Rightarrow 2\sec(\theta) = x$
$\Rightarrow 2\sec(\theta)\tan(\theta)\,d\theta = dx$

choose
$\tan(\theta) = \frac{\sqrt{x^2-4}}{2}$
$\Rightarrow 2\tan(\theta) = \sqrt{x^2-4}$

$$= \int \frac{(2\sec(\theta))^3\,\cancel{2}\sec(\theta)\cancel{\tan(\theta)}}{\cancel{2\tan(\theta)}}\,d\theta$$

$$= 8\int \sec^4(\theta)\,d\theta$$

$$= 8\int \sec^2(\theta)\,\sec^2(\theta)\,d\theta$$

RESERVED

$$= 8\int [\tan^2(\theta)+1]\,\sec^2(\theta)\,d\theta$$

Let $u = \tan(\theta)$
$du = \sec^2(\theta)\,d\theta$

$$= 8\int [u^2+1]\,du$$

$$= 8\left[\frac{u^3}{3}+u\right]+C$$

$$= 8\left[\frac{\tan^3(\theta)}{3}+\tan(\theta)\right]+C$$

$$= \boxed{8\left[\frac{1}{3}\left(\frac{\sqrt{x^2-4}}{2}\right)^3+\frac{\sqrt{x^2-4}}{2}\right]+C}$$

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(8). $\int \frac{x-17}{x^2+x-6}\,dx$

PAR FRAC

$\frac{x-17}{(x+3)(x-2)} = \frac{A}{(x+3)} + \frac{B}{(x-2)}$

$\Rightarrow x-17 = A(x-2) + B(x+3)$

$\Rightarrow$ Let $x=2$

$\rightarrow 2-17 = A(0) + B(2+3)$

$\rightarrow B = -5$

Let $x=-3$

$\rightarrow -3-17 = A((-3)-2) + B(0)$

$\rightarrow A = 4$

so the integral is:

$= \int \left[\frac{4}{x+3} - \frac{5}{x-2} \right] dx$

$= \boxed{4 \ln|x+3| - 5 \ln|x-2| + C}$

(9). $\int \frac{4x^2-2x+9}{x^3+3x}\,dx$

PAR FRAC:

$\frac{4x^2-2x+9}{x(x^2+3)} = \frac{A}{x} + \frac{Bx+C}{x^2+3}$

$\Rightarrow 4x^2 - 2x + 9 = A(x^2+3) + (Bx+C)(x)$

$= Ax^2 + 3A + Bx^2 + Cx$

$= (A+B)x^2 + Cx + 3A$

$\Rightarrow \begin{array}{ll} A+B &= 4 \Rightarrow B=1 \\ C &= -2 \Rightarrow C=-2 \\ 3A &= 9 \Rightarrow A=3 \end{array}$

so we have:

$= \int \left[\frac{3}{x} + \frac{x-2}{x^2+3} \right] dx$

$= \int \left[\frac{3}{x} + \frac{x}{x^2+3} - \frac{2}{x^2+3} \right] dx$

we have

$\int \frac{3}{x}\,dx = 3 \ln|x| + C_1$

$\int \frac{x}{x^2+3}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$

Let $u = x^2+3$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\ln|u| + C_2$

$= \frac{1}{2}\ln|x^2+3| + C_2$

cont.

cont.

$$\int \frac{2}{x^2+3}\,dx = 2\int \frac{1}{u^2+a^2}\,du = 2\left(\frac{1}{\sqrt{3}}\right)\tan^{-1}\left(\frac{u}{a}\right)+C_3$$

$\tan^{-1}$ $u=x$ $a=\sqrt{3}$

$du=dx$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+C_3$$

so the result is:

$$= \boxed{3\ln|x| + \frac{1}{2}\ln|x^2+3| - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+C}$$

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(10) $\int 2x \ln(x)\,dx =$

parts

$u = \ln(x)$ $\quad v = \frac{2x^2}{2} = x^2$

$du = \frac{1}{x}dx$ $\quad dv = 2x\,dx$

$$= x^2\ln(x) - \int \frac{x^2}{x}\,dx$$

$$= x^2\ln(x) - \int x\,dx$$

$$= \boxed{x^2\ln(x) - \frac{x^2}{2} + C}$$

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(11) $\lim_{x\to1^+} \dfrac{1-x+\ln(x)}{x^3-3x+2} \overset{L'H}{=} \lim_{x\to1^+} \dfrac{-1+\frac{1}{x}}{3x^2-3}$

$\overset{L'H}{=} \lim_{x\to1^+} \dfrac{-1x^{-2}}{6x}$

$= \boxed{\dfrac{-1}{6}}$

NOTE: should be

(12) $\lim_{x\to0^+} x^{(x^3)} \to 0^0$

Let $y = \lim_{x\to0^+} x^{(x^3)}$

$\Rightarrow \ln(y) = \ln\left(\lim_{x\to0^+} x^{(x^3)}\right)$

$= \lim_{x\to0^+}\left(\ln\left(x^{(x^3)}\right)\right)$

$= \lim_{x\to0^+}\left(x^3\cdot\ln(x)\right)$

$= \lim_{x\to0^+} \dfrac{\ln(x)}{x^{-3}}$

$\overset{L'H}{=} \lim_{x\to0^+} \dfrac{\frac{1}{x}}{-3x^{-4}}$

$= \lim_{x\to0^+} \dfrac{x^4\cdot\frac{1}{x}}{-3}$

$= \lim_{x\to0^+} \dfrac{x^3}{-3}$

$= 0$

SO

$\ln(y) = 0$

$\Rightarrow e^{\ln(y)} = e^0$

$\Rightarrow \boxed{y = 1}$

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13. $$\lim_{x\to 0} \frac{x^2 - x}{\tan(x)} \overset{L'H}{=} \lim_{x\to 0} \frac{2x-1}{\sec^2(x)}$$

$$= \frac{2(0)-1}{\sec^2(0)}$$

$$= \frac{-1}{1}$$

$$= \boxed{-1}$$

III

14. $$\int_{-1}^{\infty} e^{(-x)}\,dx = \lim_{b\to\infty}\left[\int_0^b e^{(-x)}\,dx\right]$$

$$= \lim_{b\to\infty}\left[-e^{(-x)}\right]_0^b$$

$$= \lim_{b\to\infty}\left[(-e^{-b}) - (-e^{(-0)})\right]$$

$$= 0 + e^0$$

$$= \boxed{1}$$

(15) $\int_1^3 \frac{1}{(x-2)^2}dx \to$ NOTE Asymptote at $x=2$

$$\Rightarrow \int_1^3 \frac{1}{(x-2)^2}dx = \int_1^2 \frac{1}{(x-2)^2}dx + \int_2^3 \frac{1}{(x-2)^2}dx$$

$$= \lim_{b\to 2^-} \int_1^b \frac{1}{(x-2)^2}dx + \lim_{a\to 2^+} \int_a^3 \frac{1}{(x-2)^2}dx$$

Note: $\int_a^b \frac{1}{(x-2)^2}dx = \int_a^b (x-2)^{-2}dx$

Let $u = x-2$

$du = dx$

$$= \int_c^d u^{-2}du$$

$$= \left[\frac{u^{-1}}{-1}\right]_c^d$$

$$= -(x-2)^{-1}\Big]_a^b$$

$$= \frac{-1}{(x-2)}\Big]_a^b$$

So we have

$$= \lim_{b\to 2^-}\left[\frac{-1}{(x-2)}\right]_1^b + \lim_{b\to 2^+}\left[\frac{-1}{(x-2)}\right]_a^3$$

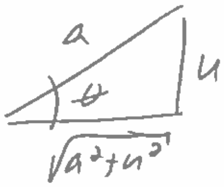
$$= \lim_{b\to 2^-}\left[\left(\frac{-1}{b-2}\right)-\left(\frac{-1}{(1-2)}\right)\right] + \lim_{b\to 2^+}\left[\left(\frac{-1}{(3-2)}\right)-\left(\frac{-1}{a-2}\right)\right]$$

$$= [\infty - 1] + [-1+\infty]$$

$$= \boxed{\infty}$$

IIII

(16) $\displaystyle\int \frac{u^2}{\sqrt{a^2+u^2}}\,du =$



Choose $\tan(\theta) = \frac{u}{a}$ choose $\sec(\theta) = \frac{\sqrt{a^2+u^2}}{a}$

$\Rightarrow a\tan(\theta) = u$ $a\sec(\theta) = \sqrt{a^2+u^2}$

$\Rightarrow a\sec^2(\theta)\,d\theta = du$

$$= \int \frac{a^2\tan^2(\theta)\cdot \cancel{a}\,\cancel{\sec^2(\theta)}^{\sec(\theta)}}{\cancel{a}\,\cancel{\sec(\theta)}}\,d\theta$$

$$= a^2\int \tan^2(\theta)\sec(\theta)\,d\theta$$

$$= a^2\int [\sec^2(\theta) - 1]\sec(\theta)\,d\theta$$

$$= a^2\int [\sec^3(\theta) - \sec(\theta)]\,d\theta$$

$$= a^2\left[\frac{1}{2}\left[\sec(\theta)\tan(\theta) + \ln|\sec(\theta)+\tan(\theta)|\right] - \ln|\sec(\theta)+\tan(\theta)|\right] + C$$

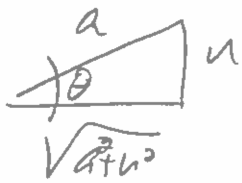
$$= a^2\left[\frac{1}{2}\sec(\theta)\tan(\theta) - \frac{1}{2}\ln|\sec(\theta)+\tan(\theta)|\right] + C$$

$$= \frac{a^2}{2}\left(\frac{u}{a}\right)\left(\frac{\sqrt{a^2+u^2}}{a}\right) - \frac{a^2}{2}\ln\left|\frac{u}{a} + \frac{\sqrt{a^2+u^2}}{a}\right| + C$$

$$= \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln\left(u+\sqrt{a^2+u^2}\right) - \ln|a| + C$$

$$= \boxed{\frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln\left(u+\sqrt{a^2+u^2}\right) + C}$$

(17) $$\int \frac{du}{u\sqrt{a^2+u^2}} =$$



choose $\tan(\theta) = \frac{u}{a}$

choose $\sec(\theta) = \frac{\sqrt{a^2+u^2}}{a}$

$\Rightarrow a\tan(\theta) = u$

$\Rightarrow a\sec(\theta) = \sqrt{a^2+u^2}$

$\Rightarrow a\sec^2(\theta)\,d\theta = du$

$$= \int \frac{\cancel{a}\,\overset{\sec(\theta)}{\cancel{\sec^2(\theta)}}}{a\tan(\theta)\,\cancel{a\sec(\theta)}}\,d\theta$$

$$= \int \frac{\sec(\theta)}{a\tan(\theta)}\,d\theta$$

$$= \frac{1}{a}\int \frac{\cancel{\cos(\theta)}}{\cancel{\cos(\theta)}\sin(\theta)}\,d\theta$$

$$= \frac{1}{a}\int \frac{1}{\sin(\theta)}\,d\theta$$