

I _____

① ② $f(x) = 5x^5 + 3x^{-3} - 12x^{1/2} \Rightarrow f'(x) = [25x^4 - 9x^{-4} - 6x^{-1/2}]$

③ $f(x) = \tan(3x) - \cos(-x) \Rightarrow f'(x) = [\sec^2(3x)(3) - (-\sin(-x)(-1))]$
 OR $[3\sec^2(3x) - \sin(-x)]$

④ $f(x) = x^4 \sec(x) \Rightarrow f'(x) = [x^4 \sec(x) \tan(x) + \sec(x)(4x^3)]$

⑤ $f(x) = (-x^{-5} - x^{-2})^{-3} \Rightarrow f'(x) = (-3)(-x^{-5} - x^{-2})^{-4} (5x^{-6} + 2x^{-3})$

⑥ $f(x) = \frac{x^{1/3} + \cot(x)}{4x} \Rightarrow f'(x) = \frac{(4x)(\frac{1}{3}x^{-2/3} - \csc^2(x)) - (x^{1/3} + \cot(x))(4)}{(4x)^2}$

⑦ $f(x) = \frac{\sin^2(x^2)}{(\sin(x^2))^2} \Rightarrow f'(x) = [2 \sin(x^2) (\cos(x^2)(2x))]$
 OR $[4x \sin(x^2) \cos(x^2)]$

⑧ $f(x) = (2 - x^3)^{1/4} \Rightarrow f'(x) = [\frac{1}{4}(2 - x^3)^{-3/4} (-3x^2)]$

⑨ $f(x) = \frac{x^2 + 5}{x^5 + 2} \Rightarrow f'(x) = \frac{(x^2 + 5)(2x) - (x^5 + 2)(5x^4)}{(x^5 + 2)^2}$

III

$$10. f(x) = 2x^4 + 3x^{-2} - 12x$$

$$\Rightarrow f'(x) = 8x^3 - 6x^{-3} - 12 \Rightarrow f''(x) = \boxed{24x^2 + 18x^{-4}}$$

$$11. f(x) = x + \tan(2x)$$

$$\Rightarrow f'(x) = 1 + \sec^2(2x)(2)$$

$$= 1 + 2\sec^2(2x)$$

$$= 1 + 2(\sec(2x))^2$$

$$\Rightarrow f''(x) = 2(2)(\sec(2x))(\sec(2x)\tan(2x))$$

$$= \boxed{8\sec^2(2x)\tan(2x)}$$

III

$$12. x^3 + 5x^2y^2 = 3y$$

$$\Rightarrow 3x^2 + 5x^2(2y\frac{dy}{dx}) + (y^2)(10x) = 3\frac{dy}{dx}$$

$$\Rightarrow 3x^2 + 10x^2y\frac{dy}{dx} + 10xy^2 = 3\frac{dy}{dx}$$

$$\Rightarrow 10x^2y\frac{dy}{dx} - 3\frac{dy}{dx} = -10xy^2 - 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \boxed{\frac{-10xy^2 - 3x^2}{10x^2y - 3}}$$

$$13. 3y - \cos(x+4y) = 2x$$

$$\Rightarrow 3\frac{dy}{dx} + \sin(x+4y)(1+4\frac{dy}{dx}) = 2$$

$$\Rightarrow 3\frac{dy}{dx} + \sin(x+4y) + 4\sin(x+4y)\frac{dy}{dx} = 2$$

$$\Rightarrow 3\frac{dy}{dx} + 4\sin(x+4y)\frac{dy}{dx} = 2 - \sin(x+4y)$$

$$\Rightarrow \frac{dy}{dx} = \boxed{\frac{2 - \sin(x+4y)}{3 + 4\sin(x+4y)}}$$

I 14. Find $\left| \frac{dy}{dt} \right| =$

Given $\frac{dx}{dt} = 2$ $z = 5$

NOTE! $\frac{dz}{dt} = -1$ $y = 4$

We have:

$$x^2 + y^2 = z^2$$

$$\Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$\hookrightarrow$ must find x

$$\Rightarrow x^2 + (4)^2 = (5)^2$$

$$\Rightarrow x^2 = 9 \Rightarrow x = 3$$

so

$$(3)(2) + (4) \frac{dy}{dt} = (5)(-1)$$

$$\Rightarrow 6 + 4 \frac{dy}{dt} = -5$$

$$\Rightarrow \frac{dy}{dt} = \boxed{-\frac{11}{4}} \text{ m/s}$$

15. Find $\frac{dA}{dt}$

given $\frac{dV}{dt} = 72\pi$

$V = 36\pi$

WE HAVE

$$V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$\hookrightarrow$ NEED VALUE FOR r :

$$36\pi = \frac{4}{3}\pi r^3$$

$$\Rightarrow r^3 = \frac{36\pi}{\frac{4}{3}\pi} = 27$$

$$\Rightarrow r = 3$$

so $A = 4\pi r^2$

$$\Rightarrow \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$= 8(\pi)(3)(2)$$

$$= \boxed{48\pi} \text{ cm}^2/\text{sec}$$

so we have:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow 72\pi = 4\pi(3)^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{72\pi}{36\pi} = \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = 2$$