SOLUTIONS

I. $W = 320 \Rightarrow \int_0^{16} kx\,dx = 320$

$$\Rightarrow \left[\frac{kx^2}{2}\right]_0^{16} = 320$$

$$\Rightarrow \left[\left(\frac{k(16)^2}{2}\right) - 0 = 320\right.$$

$$\Rightarrow 256k = 640$$

$$\Rightarrow k = \frac{640}{256} = 2.5$$

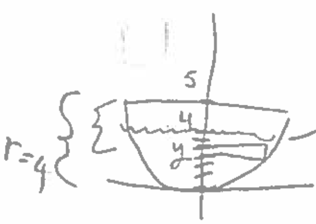
so $F(x) = 2.5x$ so $\boxed{W = \int_0^{13} 2.5x\,dx}$

$$= \left[\frac{2.5x^2}{2}\right]_0^{13}$$

$$= \left(\frac{2.5(13)^2}{2}\right) - 0$$

$$= \boxed{211.25}$$

II.



EQUATION OF CIRCLE $\Rightarrow (x-0)^2 + (y-5)^2 = 25$ (radius)^2

Center $= (0,5)$
radius $= 5$

$$\Rightarrow x^2 + (y-5)^2 = 25$$

$$\Rightarrow x = \pm\sqrt{25-(y-5)^2}$$

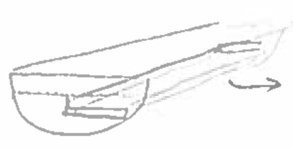
choose the positive

$$\Rightarrow x = \sqrt{25-(y-5)^2}$$

We may use symmetry: find the work for the right-half and double it.

$\Delta W =$ (weight of water)(distance lifted)

NOTE THE SHAPE OF EACH "SLICE" of WATER:



volume of the slice is (length)(width)(thickness)
$= (20)(\sqrt{25-(y-5)^2})(\Delta y)$

cont.

(II) CONT: ⇓ pg two

$$\Delta W = \left(\text{weight of water}\right)\left(\text{distance Lifted}\right)$$

$$= \left(\text{density}\right)\boxed{\left(\text{volume}\right)}\left(\text{distance lifted}\right)$$

$$= (\delta)(20)\left(\sqrt{25-(y-5)^2}\right)(\Delta y)(5-y)$$

so

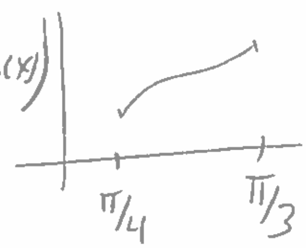
$$W = \boxed{\delta \int_0^4 20\left(\sqrt{25-(y-5)^2}\right)(5-y)\,dy}$$

NOTE: LIMITS OF INTEGRATION ARE FOUND BY PARTITIONING THE <u>LIQUID</u>, NOT THE ENTIRE TANK.

(III) $y = \ln(\sec(x)) + 4$ on $\pi/4 \le x \le \pi/3$

$$y' = \frac{1}{\sec(x)}\left(\sec(x)\tan(x)\right)$$

$$= \tan(x)$$



so

$$s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$$

$$= \int_{\pi/4}^{\pi/3} \sqrt{1+[\tan(x)]^2}\,dx$$

$$= \int_{\pi/4}^{\pi/3} \sqrt{\sec^2(x)}\,dx$$

$$= \int_{\pi/4}^{\pi/3} \sec(x)\,dx$$

$$= \Big[\ln|\sec(x)+\tan(x)|\Big]_{\pi/4}^{\pi/3}$$

$$= \Big[\left(\ln|\sec(\pi/3)+\tan(\pi/3)|\right) - \left(\ln|\sec(\pi/4)+\tan(\pi/4)|\right)\Big]$$

$$= \boxed{\Big[\left(\ln|2+\sqrt{3}|\right) - \left(\ln\left|\tfrac{2}{\sqrt{2}}+1\right|\right)\Big]}$$

III ①. $F_1 = k_1 X$ $\quad\quad$ $F_2 = k_2 X$

$\Rightarrow 96 = k_1(4)$ $\quad\quad$ $\Rightarrow 112 = k_2(7)$

$\Rightarrow k_1 = 24$ $\quad\quad$ $\Rightarrow k_2 = 16$

$\Rightarrow F_1 = 24X$ $\quad\quad$ $\Rightarrow F_2 = 16X$

so Ⓐ $W_A = \int_2^4 24x\,dx$

$= \left[\frac{24x^2}{2}\right]_2^4$

$= (12(4)^2) - (12(2)^2)$

$= 192 - 48$

$= 144$

Ⓑ $W_B = \int_3^5 16x\,dx$

$= \left[\frac{16x^2}{2}\right]_3^5$

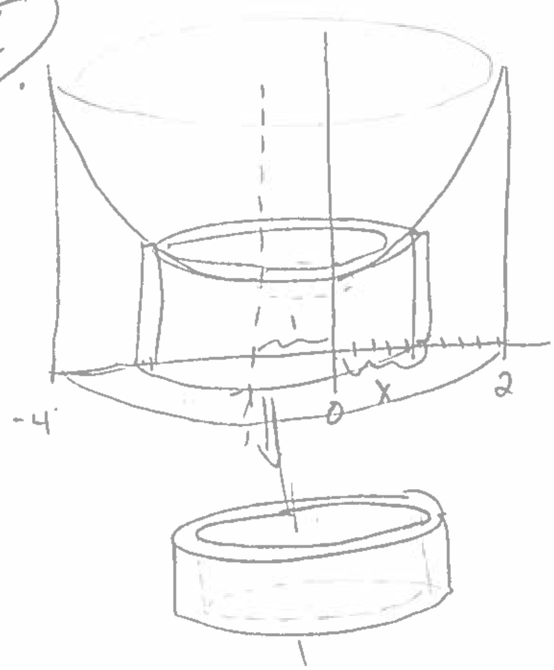
$= (8(5)^2) - (8(3)^2)$

$= 200 - 72$

$= 128$

so $\boxed{\text{Ⓐ REQUIRES MORE WORK}}$

IV.



$\Delta V = 2\pi\,(\text{RADIUS})(\text{HEIGHT})(\text{THICKNESS})$

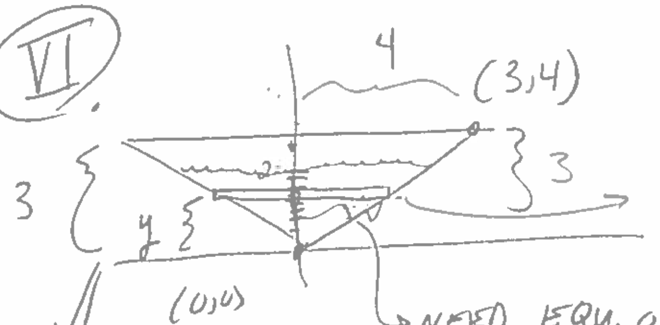
$= 2\pi\,(x-(-1))(e^x)(\Delta x)$

$= 2\pi\,(x+1)(e^x)(\Delta x)$

so

$$\boxed{V = 2\pi \int_0^2 (x+1)(e^x)\,dx}$$

VI.



distance lifted $=(3-y)$

NEED EQU. OF THIS LINE FOR THE RADIUS (AS A FUNCTION OF y!!) $\Longrightarrow$ $\left.\begin{array}{l} m=\frac{\text{rise}}{\text{run}}=\frac{3}{4} \\ y\text{-int}=y(0,0)\Rightarrow 0 \end{array}\right\} \Rightarrow y=\frac{3}{4}x$

$\Rightarrow \frac{4}{3}y = x$

So

$$\Delta W = \left(\text{Weight of Liquid}\right)\left(\text{distance Lifted}\right)$$

$$= \left(\text{density}\right)\left(\text{Volume}\right)\left(\text{distance lifted}\right)$$

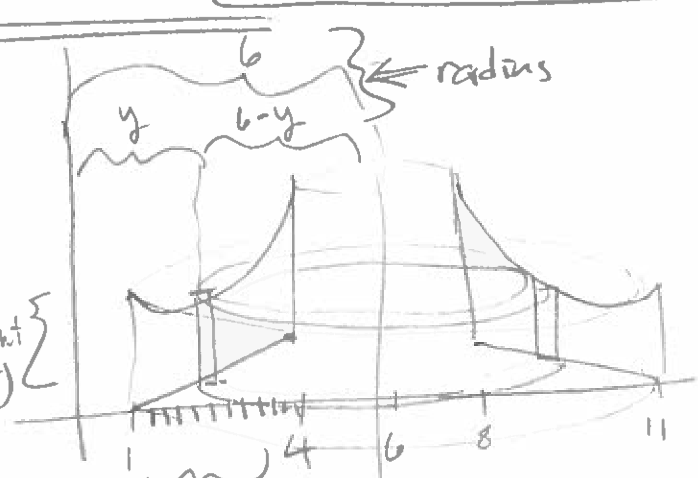
$$= (\delta)\left(\pi(\text{radius})^2(\text{thickness})\right)\left(\text{distance lifted}\right)$$

$$= \delta\left(\pi\left(\tfrac{4}{3}y\right)^2(\Delta y)\right)(3-y)$$

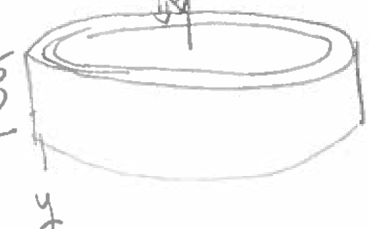
So

$$W = \boxed{\delta\pi\int_0^2 \left(\tfrac{4}{3}y\right)^2(3-y)\,dy}$$

VII.



partition here



height = top curve minus bottom curve.

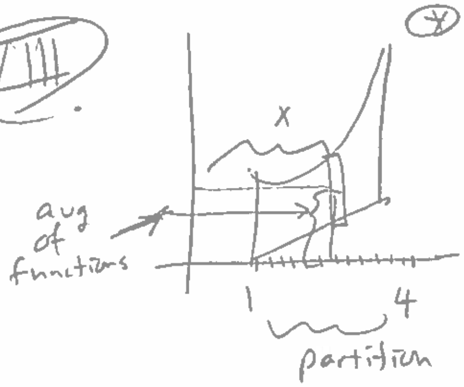
$$\Delta V = 2\pi(\text{radius})(\text{height})(\text{thickness})$$

$$= 2\pi(6-y)\left((x^2-4x+5)-\left(\tfrac{1}{3}x-\tfrac{1}{3}\right)\right)\cdot(\Delta x)$$

So

$$\boxed{V = 2\pi\int_1^4 (6-y)\left((x^2-4x+5)-\left(\tfrac{1}{3}x-\tfrac{1}{3}\right)\right)dx}$$

VIII.



*ASSUME density constant $= \rho$ *

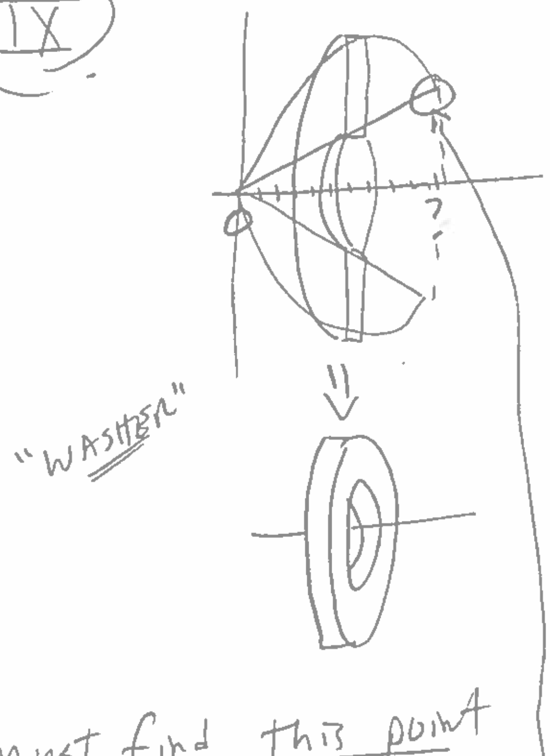
we need

$$M_x = \rho \int_1^4 \left[(x^2-4x+5) - \left(\tfrac{1}{3}x - \tfrac{1}{3}\right)\right]\left[\frac{(x^2-4x+5) + \left(\tfrac{1}{3}x-\tfrac{1}{3}\right)}{2}\right] dx$$

$$M_y = \rho \int_1^4 \left[(x^2-4x+5) - \left(\tfrac{1}{3}x - \tfrac{1}{3}\right)\right][x]\, dx$$

$$m = \rho \int_1^4 \left[(x^2-4x+5) - \left(\tfrac{1}{3}x - \tfrac{1}{3}\right)\right] dx$$

IX.



$$V = \pi \int_0^3 \left[\left(\overset{\text{Parabola}}{\text{OUTER RADIUS}}\right)^2 - \left(\overset{\text{Line}}{\text{INNER RADIUS}}\right)^2\right] dx$$

$$= \boxed{\pi \int_0^3 \left[(4x-x^2)^2 - (x)^2\right] dx}$$

Must find this point so we have the limit of integration

$\Rightarrow$ EQUATE THE TWO:

$x = 4x - x^2$

$+x^2 - 4x \quad -4x \quad +x^2$

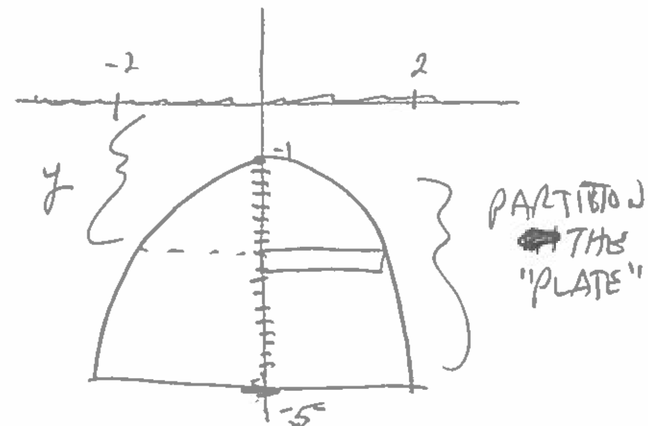
$\Rightarrow$

$x^2 - 3x = 0$

$\rightarrow x(x-3) = 0$

$\rightarrow x = 0 \quad \underline{x = 3} \rightarrow (0,0)$ & $(3,3)$

II. "INVERTED BASIC-SHAPE PARABOLA" WITH BASE 5 UNITS BENEATH SURFACE AS PICTURED" MEANS THE EQUATION IS



$$f(x) = -x^2 - 1$$

$$\Rightarrow \quad y = -x^2 - 1$$

NEED THIS AS A FUNCTION OF "y"

$$\Rightarrow \quad y+1 = -x^2$$

$$\Rightarrow \quad -y-1 = x^2$$

$$\Rightarrow \quad \pm\sqrt{-y-1} = x$$

choose the <u>positive</u>

$$\Rightarrow \quad x = \sqrt{-y-1}$$

BY SYMMETRY, WE MAY FIND THE FORCE ON ONE HALF OF THE PLATE AND DOUBLE IT.

SO

$$\Delta F = 2\begin{pmatrix}\text{Density}\\ \text{CONSTANT}\end{pmatrix}(\text{AREA})(\text{depth})$$

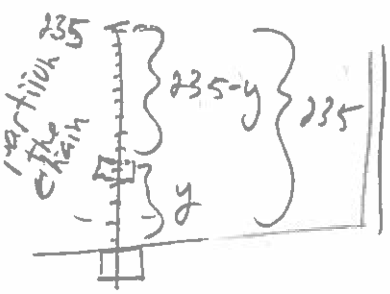
$$= 2(\rho)(\text{length})(\text{width})(\text{depth})$$

$$= 2\rho\left(\sqrt{-y-1}\right)(\Delta y)(y)$$

so

$$F = \boxed{2\rho\int_{-5}^{-1}(y)\left(\sqrt{-y-1}\right)dy}$$

VII



SO:

$$\Delta W = \underbrace{\begin{pmatrix}\text{weight of}\\ \text{partition}\end{pmatrix}}(\text{distance lifted})$$

$$= \begin{pmatrix}\text{weight}\\ \text{of}\\ \text{chain}\end{pmatrix}\begin{pmatrix}\text{length of}\\ \text{partition}\end{pmatrix}(\text{distance lifted})$$

$$= (17)(\Delta y)(235-y)$$

part A

so

$$W = \int_0^{235}(17)(235-y)\,dy$$

$$= \int_0^{235}(3995-17y)\,dy$$

see NEXT pg

CONT:

VII

$$= \left[3995y - \frac{17y^2}{2}\right]_0^{235}$$

$$= \left[\left(3995(235) - \frac{17(235)^2}{2}\right) - (0-0)\right]$$

$$= 938{,}825 - 469{,}412.5$$

$$= \boxed{469{,}412.5}$$

Part B

$$\text{TOTAL WORK} = \begin{pmatrix}\text{WORK}\\ \text{FOR}\\ \text{CHAIN}\end{pmatrix} + \begin{pmatrix}\text{WORK}\\ \text{FOR}\\ \text{WEIGHT}\end{pmatrix}$$

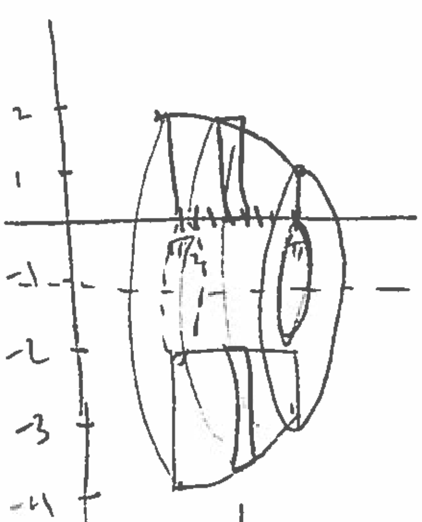
$$= 469412.5 + (10)(235)$$

$$= 469{,}412.5 + 2350$$

$$= \boxed{471762.5}$$

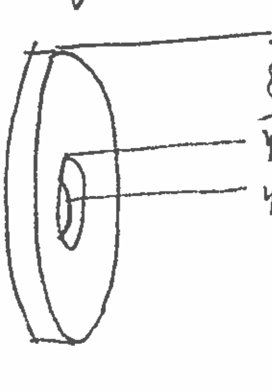
} I HAVE NO IDEA WHY I USED SUCH GOOFY NUMBERS THAT SEMESTER

#13



$$\text{so } V = \pi \int_{\pi/2}^{\pi} \left[\begin{pmatrix}\text{outer}\\ \text{radius}\end{pmatrix}^2 - \begin{pmatrix}\text{inner}\\ \text{radius}\end{pmatrix}^2\right] dx$$

$$= \boxed{\pi \int_{\pi/2}^{\pi} \left[(2+\sin(x))^2 - (1)^2\right] dx}$$



} inner radius ⇓ always is "1"

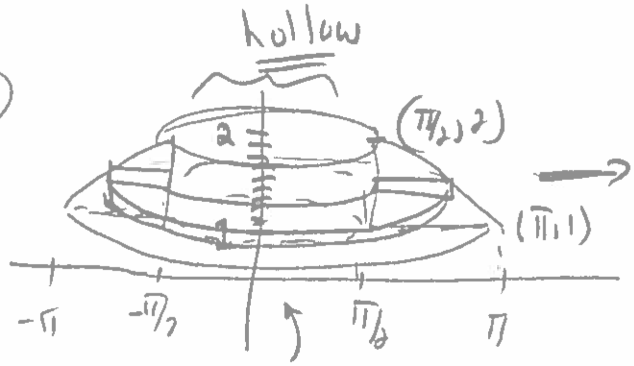
$$\begin{pmatrix}\text{outer}\\ \text{radius}\end{pmatrix} = \overset{\text{top}}{(1+\sin(x))} - \overset{\text{bottom}}{(-1)}$$

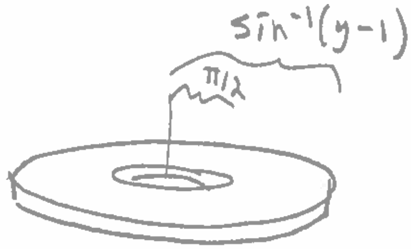
$$= 1+\sin(x)+1$$

$$= \underline{2+\sin(x)}$$

(B)

Pg 8ight





partition
y-axis

NOTE THAT THE INNER RADIUS IS CONSTANT at $\frac{\pi}{2}$

The outer radius is the function value, but we need a function of "y"

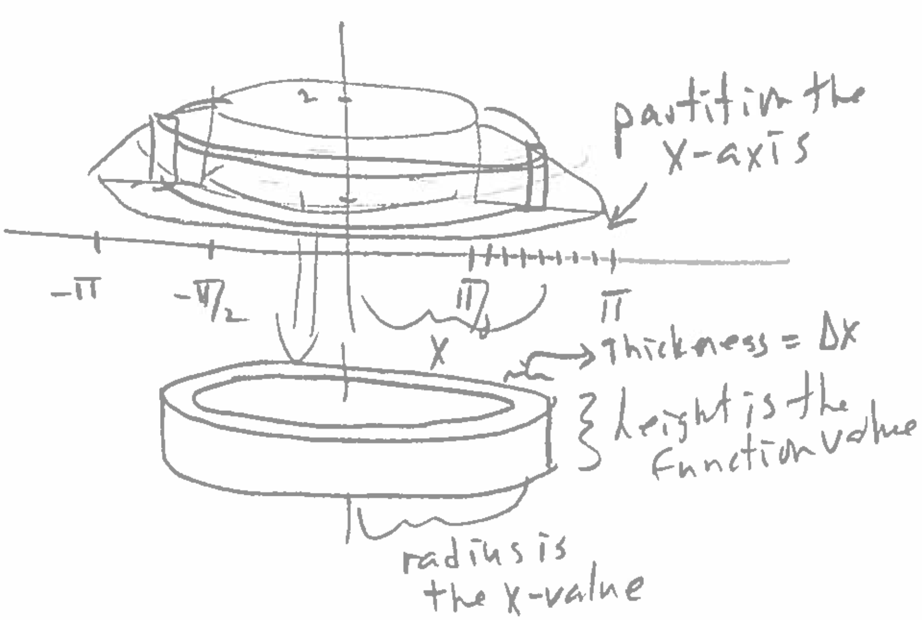
$\Rightarrow y = 1 + \sin(x) \Rightarrow y - 1 = \sin(x) \Rightarrow \underline{\sin^{-1}(y-1) = x}$

so

$$V = \pi \int_1^2 \left[\begin{pmatrix} \text{INNER} \\ \text{RADIUS} \end{pmatrix}^2 - \begin{pmatrix} \text{outer} \\ \text{radius} \end{pmatrix}^2 \right] dy$$

$$= \boxed{\pi \int_1^2 \left[\left(\frac{\pi}{2}\right)^2 - \left(\sin^{-1}(y-1)\right)^2 \right] dy}$$

OR you may use the shell method:



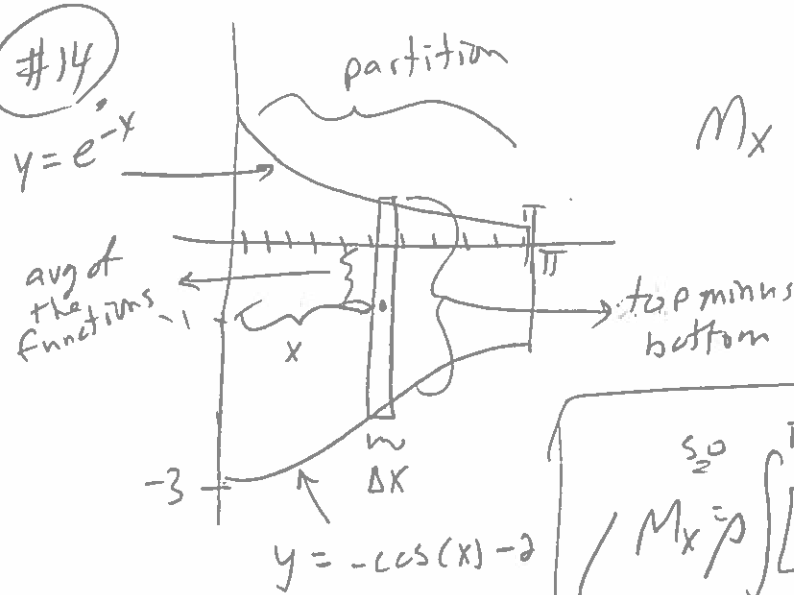
so $\Delta V = (2\pi)(\text{radius})(\text{height})(\text{thickness})$

$= 2\pi(x)(1+\sin(x))(\Delta x)$

so

$$\boxed{V = \int_{\pi/2}^{\pi} 2\pi x \left(1 + \sin(x)\right) dx}$$

#14



pg 9 me

$$M_x = \rho \int_0^{\pi} \left[(e^{-x}) - (-\cos(x)-2)\right]\left[(e^{-x})+(-\cos\right.$$

so

$$M_x = \rho \int_0^{\pi} \left[(e^{-x}) - (-\cos(x)-2)\right]\left[\frac{(e^{-x})+(-\cos(x)-2)}{2}\right] dx$$

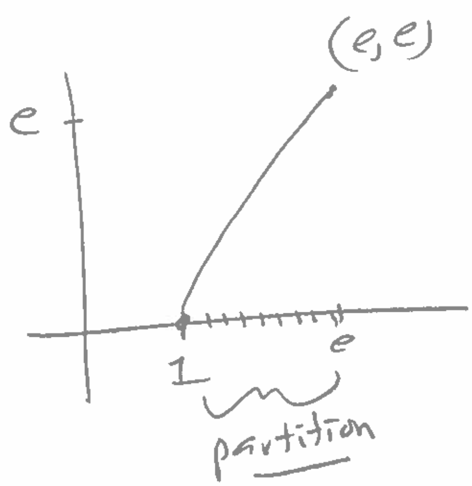
moments

$$M_y = \rho \int_0^{\pi} \left[(e)^{-x} - (-\cos(x)-2)\right](x)\,dx$$

Total mass

$$M = \rho \int_0^{\pi} \left[(e^{-x}) - (-\cos(x)-2)\right] dx$$

#15



$y = f(x) = x\ln(x)$

$\Rightarrow f'(x) = x \cdot \frac{1}{x} + \ln(x) + 1$

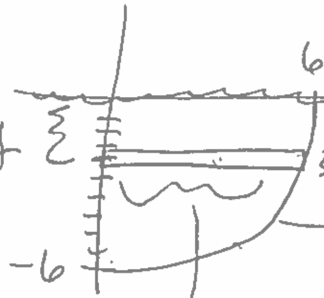
$= 1 + \ln(x)$

so

$$S = \int_1^e \sqrt{1 + [f'(x)]^2}\,dx$$

$$= \boxed{\int_1^e \sqrt{1 + (1+\ln(x))^2}\,dx}$$

16.



$\to$ width $= \Delta y$

depth is the y-coordinate

length is the function value $x = \sqrt{36-y^2}$

$\to x^2+y^2=6^2$

$\Rightarrow x^2+y^2=36$

$\Rightarrow x = \pm\sqrt{36-y^2}$

choose $x=\sqrt{36-y^2}$

$\Delta F = (\text{density})(\text{depth})(\text{Area})$

$= (w)(y)(\text{length})(\text{width})$

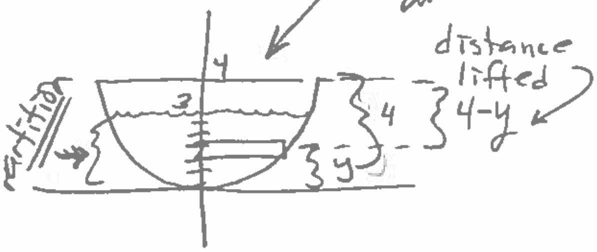
$= (w)(y)(\sqrt{36-y^2})(\Delta y)$

so total force is the integral

$$\boxed{F = w\int_{-6}^{0} y\left(\sqrt{36-y^2}\right)dy}$$

(II) ALTERNATE

we find half and double!



So $\Delta W = 2 \cdot (\text{weight of water})(\text{distance lifted})$

$= 2(\text{density})(\text{Volume of the "slab"})(\text{distance lifted})$

$= 2(\delta)(\text{length})(\text{width})(\text{thickness})(4-y)$

$= 2\delta(20)\left(\sqrt{16-(y-4)^2}\right)(\Delta y)(4-y)$

EQUATION OF THE CIRCLE IS $(x-0)^2+(y-4)^2=16$

$\Rightarrow x=\sqrt{16-(y-4)^2}$

SO we have:

$$W = \boxed{2\delta\int_0^3 20\left(\sqrt{16-(y-4)^2}\right)(4-y)\,dy}$$

|| See also the OTHER SOLUTION ||