SOLUTIONS

I

① $f(x) = 4x^5 + 5x^4 \Rightarrow f'(x) = \boxed{20x^4 + 20x^3}$

② $f(x) = x^{1/2} + x^{-1/3} \Rightarrow f'(x) = \boxed{\tfrac{1}{2}x^{-1/2} - \tfrac{1}{3}x^{-4/3}}$

③ $f(x) = \tan(x) - \cos(x^3) \Rightarrow f'(x) = \boxed{\sec^2(x) - (-\sin(x^3)(3x^2))}$

④ $f(x) = \dfrac{x^{-3} - x}{\sin(x)} \Rightarrow f'(x) = \boxed{\dfrac{(\sin(x))(-3x^{-4} - 1) - (x^{-3} - x)(\cos(x))}{(\sin(x))^2}}$

⑤ $f(x) = x^4 \tan(x) \Rightarrow f'(x) = \boxed{(x^4)(\sec^2(x)) + (\tan(x))(4x^3)}$

⑥ $f(x) = (x^{-3} - x^{-2})^{-1} \Rightarrow f'(x) = \boxed{-(x^{-3} - x^{-2})^{-2}(-3x^{-4} + 2x^{-3})}$

⑦ $f(x) = \dfrac{\sin(x)}{x^2} \Rightarrow f'(x) = \boxed{\dfrac{(x^2)(\cos(x)) - (\sin(x))(2x)}{(x^2)^2}}$

⑧ $f(x) = -x\csc(x) - \cos(x)$

$\Rightarrow f'(x) = \boxed{(-x)(-\csc(x)\cot(x)) + (\csc(x))(-1) - (-\sin(x))}$

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9. $f(x) = (1+\cot(x))^{1/2}$

$\Rightarrow f'(x) = \boxed{\tfrac{1}{2}(1+\cot(x))^{-1/2}(0-\csc^2(x))}$

10. $f(x) = \left(\frac{x-2}{x-4}\right)^3 \Rightarrow f'(x) = \boxed{3\left(\frac{x-2}{x-4}\right)^2\left(\frac{(x-4)(1)-(x-2)(1)}{(x-4)^2}\right)}$

II

14. $f(x) = x^2 + \cos^2(x) \Rightarrow f'(x) = 2x + 2\cos(x)(-\sin(x))$

$= 2x - 2\sin(x)\cos(x)$

$\Rightarrow f''(x) = \boxed{2 - [2\sin(x)(-\sin(x)) + (\cos(x))(2\cos(x))]}$

OR if you wish

$= \boxed{2 + 2\sin^2(x) - 2\cos^2(x)}$

15. $f(x) = (x^2+2)^5 \Rightarrow f'(x) = 5(x^2+2)^4(2x)$

$= 10x\cdot(x^2+2)^4$

$\Rightarrow f''(x) = \boxed{(10x)(4(x^2+2)^3(2x)) + (x^2+2)^4(10)}$

or, if you wish

$= \boxed{80x^2(x^2+2)^3 + 10(x^2+2)^4}$

NOTE: YOU NEED NOT SIMPLIFY ON THE EXAM. I DID THESE JUST FOR "FUN"!

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III

(16) $x^2 + 3xy + y^3 = 10$ Product rule!

$\Rightarrow 2x + (3x)\left(\frac{dy}{dx}\right) + (y)(3) + 3y^2\frac{dy}{dx} = 0$

$\Rightarrow 3x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -2x - 3y$

$\Rightarrow \frac{dy}{dx} = \boxed{\frac{-2x-3y}{3x+3y^2}}$

(17). $\cos(x+2y) = x + 2y$

$\Rightarrow -\sin(x+2y)\left(1+2\frac{dy}{dx}\right) = 1 + 2\frac{dy}{dx}$

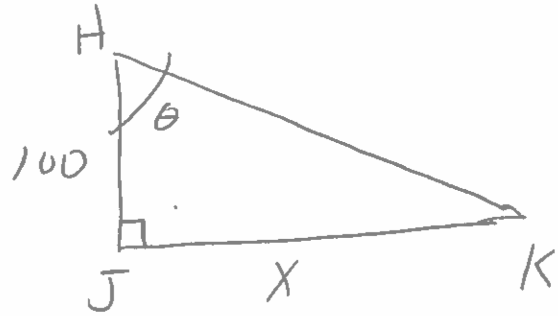
distribute

$\Rightarrow -\sin(x+2y) - 2\sin(x+2y)\frac{dy}{dx} = 1 + 2\frac{dy}{dx}$

$\Rightarrow -2\sin(x+2y)\frac{dy}{dx} - 2\frac{dy}{dx} = 1 + \sin(x+2y)$

$\Rightarrow \frac{dy}{dx} = \boxed{\frac{1+\sin(x+2y)}{-2\sin(x+2y)-2}}$

(18)



Find $\boxed{\frac{d\theta}{dt} =}$

Given $\frac{dx}{dt} = 3000$

$x = 142.85$

We have

$\tan(\theta) = \frac{x}{100}$

~~we re~~

$\Rightarrow 100\tan(\theta) = x$

differential count is ok

$\Rightarrow 100\sec^2(\theta)\frac{d\theta}{dt} = \frac{dx}{dt}$

↳ we need θ $\Rightarrow$ If $x = 142.85 \Rightarrow \tan(\theta) = \frac{142.85}{100}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{142.85}{100}\right)$

$= 55.0$ Approx

So

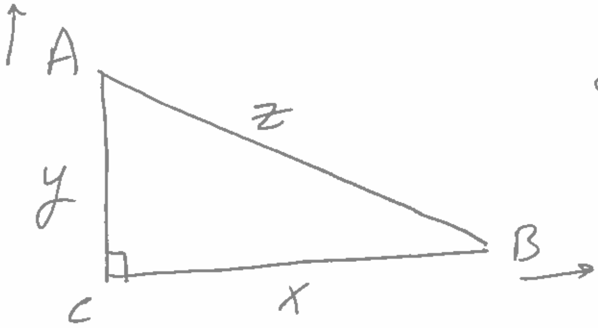
$\Rightarrow 100\sec^2(55^\circ)\frac{d\theta}{dt} = 3000$

$\Rightarrow \frac{d\theta}{dt} = \frac{3000}{100\sec^2(55)}$

$= \frac{3000\cos^2(55)}{100}$

$= \boxed{9.87^\circ \text{ deg}/\text{min}}$

19.



find $\boxed{\dfrac{dz}{dt} =}$

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Given: $\dfrac{dy}{dt} = 45 \text{ f/m}$

$\dfrac{dx}{dt} = 60 \text{ f/m}$

$t = 2$

$\Rightarrow$ we have

$x^2 + y^2 = z^2$

$\Rightarrow$ differential count is ok

$\Rightarrow 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 2z\dfrac{dz}{dt}$

$\Rightarrow$ divide thru to clear out the "2's"

$\Rightarrow x\dfrac{dx}{dt} + y\dfrac{dy}{dt} = z\dfrac{dz}{dt}$

we need values for x, y and z.

$x = 60 \text{ f/m}$ for 2 min $= 120$ ft

$y = 45 \text{ f/m}$ for 2 min $= 90$ ft

from pythagoras

$(120)^2 + (90)^2 = z^2$

$\Rightarrow 14400 + 8100 = z^2$

$\Rightarrow z = 150$

$\Rightarrow$ THUS WE HAVE

$(120)(60) + (90)(45) = (150)\dfrac{dz}{dt}$

$\Rightarrow \dfrac{7200 + 4050}{150} = \dfrac{dz}{dt}$

$\Rightarrow \boxed{75 \text{ f/m} = \dfrac{dz}{dt}}$

20. Find $\boxed{\dfrac{dV}{dt} =}$

given $h = \sqrt{3}\, r$

$\dfrac{dL}{dt} = 120 \text{ cm}^2/\text{min}$

$r = 10 \text{ cm}$

We have:

$V = \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \frac{\pi}{3} r^2 (\sqrt{3} r)$

$\Rightarrow V = \frac{\pi\sqrt{3}}{3} r^3$

$\Rightarrow$ We need $\frac{dr}{dt}$ which we can find from the Lateral Area info

find $\frac{dr}{dt}$

we have:

$L = \pi r \sqrt{r^2 + h^2}$

$\Rightarrow L = \pi r \sqrt{r^2 + (\sqrt{3} r)^2}$

$= \pi r \sqrt{r^2 + 3r^2}$

$= \pi r \sqrt{4r^2}$

$= 2\pi r^2$

so

$\Rightarrow L = 2\pi r^2$

$\frac{dL}{dt} = 4\pi r \frac{dr}{dt}$

$\Rightarrow 120 = 4\pi (10) \frac{dr}{dt}$

$\Rightarrow \frac{120}{40\pi} = \frac{3}{\pi} = \frac{dr}{dt}$

TAKE THIS VALUE BACK TO THE MAIN PROBLEM

we now have

$\frac{dr}{dt} = \frac{3}{\pi}$

so

$\Rightarrow \frac{dV}{dt} = 3\left(\frac{\pi\sqrt{3}}{3}\right) r^2 \frac{dr}{dt}$

$\Rightarrow \frac{dV}{dt} = \pi\sqrt{3} (10)^2 \left(\frac{3}{\pi}\right)$

$= (3\sqrt{3})(100)$

$= \boxed{300\sqrt{3} \text{ cm}^3/\text{min}}$