SOLUTIONS CALC II
EXAM II SPRING 2019

I.

SPRING #1

$$\int_0^4 kx\,dx = 16$$

$$\Rightarrow \left[\frac{kx^2}{2}\right]_0^4 = 16$$

$$\Rightarrow k\frac{(4)^2}{2} = 16$$

$$\Rightarrow 8k = 16$$

$$\Rightarrow k = 2$$

$$\Rightarrow F_1(x) = 2x$$

SPRING #2

$$k(6) = 36$$

$$\Rightarrow k = 6$$

$$\Rightarrow F_2(x) = 6x$$

B Requires MORE WORK

(A) $$\int_2^5 2x\,dx = \left[\frac{2x^2}{2}\right]_2^5$$

$$= [(5)^2 - (2)^2]$$

$$= \boxed{21}$$

(B) $$\int_1^3 6x\,dx = \left[\frac{6x^2}{2}\right]_1^3$$

$$= [(3(3)^2) - (3(1$$

$$= 27 - 3$$

$$= \boxed{24}$$

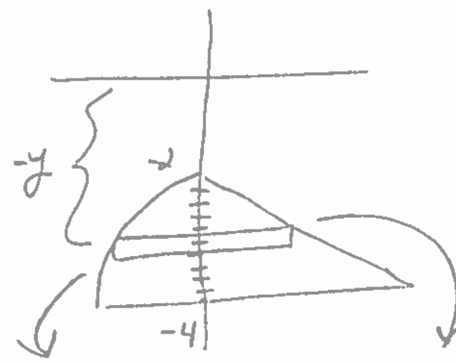
II. $\Delta F = (\text{pressure})(\text{depth})$

$= (\text{density})(\text{area})(\text{depth})$

$= (\text{density})(\text{length})(\text{width})(\text{depth})$

$= (\text{density})(\text{Right} - \text{left})(\text{width})(\text{depth})$

$$= \rho\left((-y-2) - \left(\left(\frac{y+4}{2}\right)^2 - 1\right)\right)(\Delta y)(-y)$$



$y = 2\sqrt{x+1} - 4$

$\Rightarrow y + 4 = 2\sqrt{x+1}$

$\Rightarrow \frac{y+4}{2} = \sqrt{x+1}$

$\Rightarrow \left(\frac{y+4}{2}\right)^2 = x+1$

$\Rightarrow \left(\frac{y+4}{2}\right)^2 - 1 = x$

$y = -x-2$

$\Rightarrow y+2 = -x$

$\Rightarrow -y-2 = x$

SO

$$F = \boxed{\rho\int_{-4}^{-2} (-y)\left[(-y-2) - \left(\left(\frac{y+4}{2}\right)^2 - 1\right)\right] dy}$$

III.

$f(x) = 1 + \sin(2x)$

$f'(x) = \cos(2x)(2)$

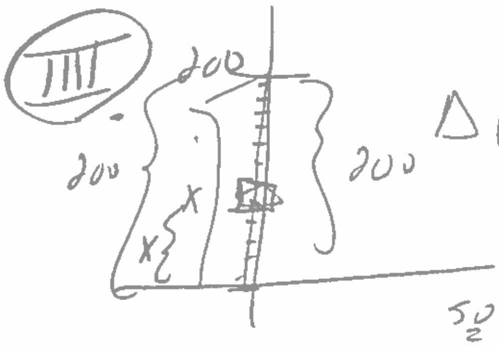
$= 2\cos(2x)$

$\Rightarrow [f'(x)]^2 = 4\cos^2(2x)$

so

$$\boxed{L = \int_0^{\pi} \sqrt{1 + 4\cos^2(2x)}\, dx}$$

IIII



$\Delta W = \Delta F(\text{distance})$

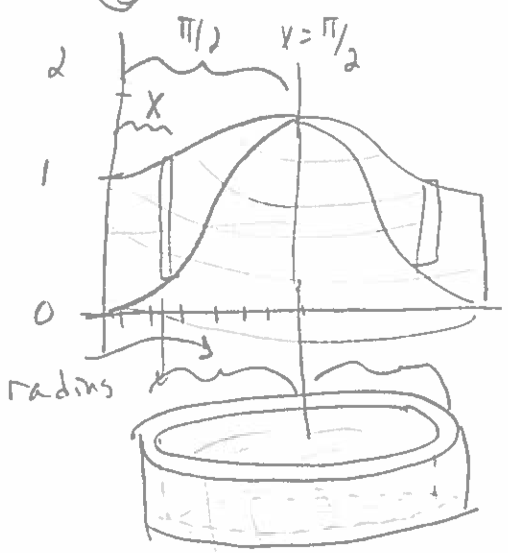
$= (\text{weight})(\text{length})(\text{distance})$

$= (6)(\Delta y)(200 - y)$

so

$$\boxed{W = \int_0^{200} (6)(200 - y)\, dy}$$

V (A) shells



note radius is Rt − Lft

$= (\pi/2 - x)$

$y = 1 + \sin(x)$

$y - 1 = \sin(x)$

$\Rightarrow x = \sin^{-1}(y-1)$

$\frac{\cos^{-1}(-y+1)}{2} = x$

so

$$V = 2\pi \int_0^{\pi/2} (\text{radius})(\text{height})\, dx$$

$$= 2\pi \int_0^{\pi/2} (\text{radius})(\text{top} - \text{bottom})\, dx$$

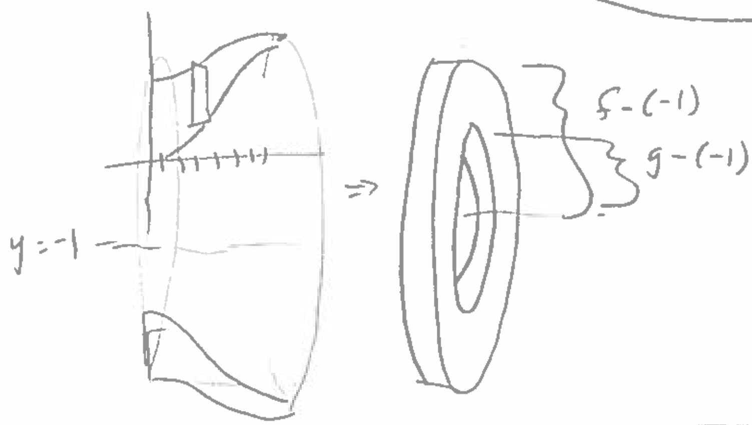
$$= \boxed{2\pi \int_0^{\pi/2} \left(\frac{\pi}{2} - x\right)\Big((1 + \sin(x)) - (1 - \cos(2x))\Big)\, dx}$$

washers

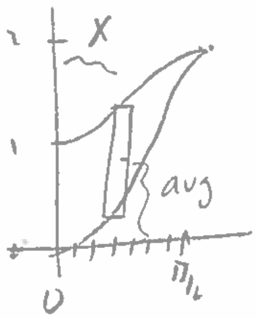
$$V = \pi \int_0^1 \left[\left(\frac{\pi}{2}\right)^2 - \left(\frac{\pi}{2} - \frac{\cos^{-1}(-y+1)}{2}\right)^2\right] dy + \pi \int_1^2 \left[\left(\frac{\pi}{2} - \sin^{-1}(y-1)\right)^2 - \left(\frac{\pi}{2} - \frac{\cos^{-1}(-y+1)}{2}\right)^2\right] dy$$

V (B) Washers

$$\boxed{V = \pi \int_0^{\pi/2} \left[\left(1+\sin(x) - (-1)\right)^2 - \left(1-\cos(2x) - (-1)\right)^2 \right] dx}$$



VI



$$M_x = \rho \int_0^{\pi/2} \left[\frac{f(x)+g(x)}{2} \right] \left[f(x) - g(x) \right] dx$$

$$M_y = \rho \int_0^{\pi/2} x \left[f(x) - g(x) \right] dx$$

$$m = \rho \int_0^{\pi/2} \left[f(x) - g(x) \right] dx$$

So

$$M_x = \rho \int_0^{\pi/2} \left[\frac{(1+\sin(x)) + (1-\cos(2x))}{2} \right] \left[(1+\sin(x)) - (1-\cos(2x)) \right] dx$$

$$M_y = \rho \int_0^{\pi/2} x \left[(1+\sin(x)) - (1-\cos(2x)) \right] dx$$

$$m = \rho \int_0^{\pi/2} \left[(1+\sin(x)) - (1-\cos(2x)) \right] dx$$

pg 4 ans

VII. $f(x) = 1 + x^{2/3}$ on $[-1, 8]$

$f'(x) = \frac{2}{3}x^{-1/3}$

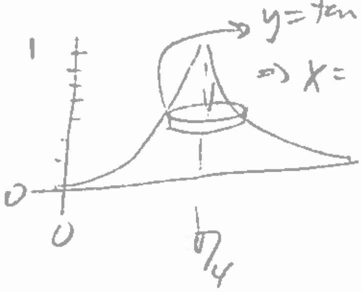
$\Rightarrow [f'(x)]^2 = \left[\frac{2}{3}x^{-1/3}\right]^2 = \frac{4x^{-2/3}}{9}$

so

$$\boxed{L = \int_{-1}^{0} \sqrt{1 + \left[\frac{4x^{-2/3}}{9}\right]}\, dx + \int_{0}^{8} \sqrt{1 + \left[\frac{4x^{-2/3}}{9}\right]}\, dx}$$

VIII. A

disk $\boxed{V = \pi \int_0^1 \left[\frac{\pi}{4} - \tan^{-1}(y)\right]^2 dy}$



$y = \tan(x)$

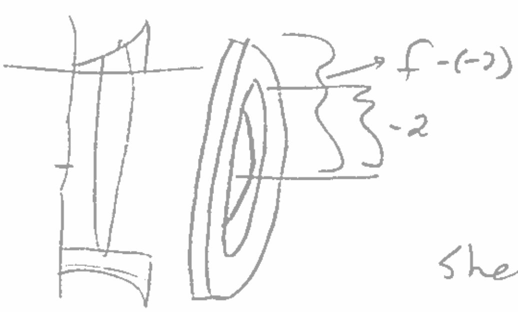
$\Rightarrow x = \tan^{-1}(y)$

shells

OR

$$\boxed{V = 2\pi \int_0^{\pi/4} \left[\frac{\pi}{4} - x\right]\left[\tan(x)\right] dx}$$

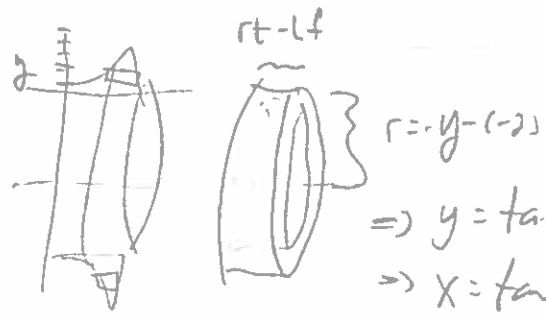
B. washer $\Rightarrow$ $\boxed{V = \pi \int_0^{\pi/4} \left[(\tan(x) - (-2))^2 - (-2)^2\right] dx}$



OR

shells

$$\boxed{V = 2\pi \int_0^1 (y - (-2))\left(\frac{\pi}{2} - \tan^{-1}(y)\right) dy}$$



$\Rightarrow y = \tan(x)$

$\Rightarrow x = \tan^{-1}(y)$

II

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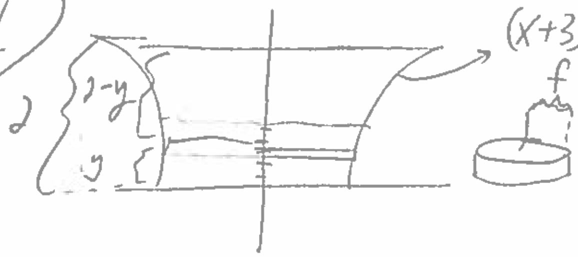

$$(x+3)^2+y^2=4$$

$$\hookrightarrow (x+3)^2 = 4-y^2$$

$$\Rightarrow x+3 = \sqrt{4-y^2}$$

$$\Rightarrow x = \sqrt{4-y^2}-3$$

$$\Delta W = (\text{Force})(\text{distance})$$

$$= (\text{density})(\text{volume})(\text{distance})$$

$$= \rho\,(\text{Area})(\text{thickness})(\text{distance})$$

$$= \rho\left(\pi\left(\sqrt{4-y^2}-3\right)^2 \Delta y\right)(2-y)$$

So

$$W = \boxed{\int_0^1 \rho\pi\left(\sqrt{4-y^2}-3\right)^2(2-y)\,dy}$$