Solutions

① $f(x) = x^4 - 4x^2 - 12$ on $[-2, 1]$

$f'(x) = 4x^3 - 8x$

Critical values

TYPE I: ($f' = 0$)

$\Rightarrow 4x^3 - 8x = 0$

$\Rightarrow 4x(x^2 - 2) = 0$

$\Rightarrow x = 2 \quad x = \pm\sqrt{2}$

$\Rightarrow$ use only $x = -\sqrt{2}$ since $x = \sqrt{2}$ is not in the interval

TYPE II (f' D.N.E.)

NONE

POLYNOMIAL

so

$f(-2) = (-2)^4 - 4(-2)^2 - 12$
$= 16 - 16 - 12 = -12$

$f(-\sqrt{2}) = (-\sqrt{2})^4 - 4(-\sqrt{2})^2 - 12$
$= 4 - 8 - 12 = -16$

$f(1) = (1)^4 - 4(1)^2 - 12$
$= 1 - 4 - 12 = -15$

so

ABS MAX at $(-2, -12)$

ABS MIN at $(-\sqrt{2}, -16)$

---

② $f(x) = \dfrac{x^2}{x-2}$ on $[3, 6]$

$\Rightarrow f'(x) = \dfrac{(x-2)(2x) - (x^2)(1)}{(x-2)^2}$

$= \dfrac{2x^2 - 4x - x^2}{(x-2)^2}$

$= \dfrac{x^2 - 4x}{(x-2)^2}$

Critical values

TYPE I ($f' = 0$)

$\Rightarrow x^2 - 4x = 0$

$\Rightarrow x(x-4) = 0$

$\cancel{x=0} \quad x = 4$

TYPE II: f' D.N.E.

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x - 2 = 0$

$\Rightarrow \cancel{x=2}$

not in the interval

so $f(3) = \dfrac{(3)^2}{(3)-2} = 9$

$f(4) = \dfrac{(4)^2}{(4)-2} = 8$

$f(6) = \dfrac{(6)^2}{(6)-2} = 9$

so

ABS MAX at $(3, 9)$ & $(6, 9)$

ABS MIN at $(4, 8)$

II. ③ $f(x) = x^4 - 32x^2 - 4$

$f'(x) = 4x^3 - 64x$

$f''(x) = 12x^2 - 64$

Critical Values

TYPE I: $f' = 0$

$4x^3 - 64x = 0$

$\Rightarrow 4x(x^2 - 16) = 0$

$\Rightarrow x = 0 \quad x = 4 \quad x = -4$

TYPE II f' D.N.E.

NONE (Polynomial)

so

$f''(0) = 12(0)^2 - 64 = -64 < 0$ Rel max

$f''(4) = 12(4)^2 - 64 = 192 - 64 = 128 > 0$ Rel min

$f''(-4) = 12(-4)^2 - 64 = 192 - 64 = 128 > 0$ Rel min

$f(0) = 0 - 0 - 4 = -4$

$f(4) = (4)^4 - 32(4)^2 - 4 = 256 - 512 - 4 = -260$

$f(-4) = (-4)^4 - 32(-4)^2 - 4 = 256 - 512 - 4 = -260$

so Rel max at $(0, -4)$

Rel min at $(4, -260)$

$(-4, -260)$

---

④ $f(x) = x + 2\cos(x)$ $[0, 3\pi]$

$f'(x) = 1 - 2\sin(x)$

$f''(x) = -2\cos(x)$

C.V. TYPE I ($f' = 0$)

$1 - 2\sin(x) = 0$

$\Rightarrow 2\sin(x) = 1$

$\Rightarrow \sin(x) = 1/2$

QI QII

$x_R = \pi/6 \Rightarrow x = \pi/6$ and $13\pi/6$

$x = 5\pi/6 \quad 17\pi/6$

TYPE II f' D.N.E.

NONE (sine function)

so $f''(\pi/6) = -2\cos(\pi/6)$ QI

$= (\text{NEG})(\text{POS})$

$= \text{Neg} \Rightarrow$ Rel Max

$f''\left(\frac{5\pi}{6}\right) = -2\cos\left(\frac{5\pi}{6}\right)$ QII

$= (\text{NEG})(\text{NEG})$

$= \text{POS} \rightarrow$ Rel MIN

$f''\left(\frac{13\pi}{6}\right) = -2\cos\left(\frac{13\pi}{6}\right)$ QI

$= (\text{NEG})(\text{POS})$

$= \text{NEG} \longrightarrow$ Rel max

$f''\left(\frac{17\pi}{6}\right) = -2\cos\left(\frac{17\pi}{6}\right)$ QII

$= (\text{NEG})(\text{NEG})$

$= \text{POS} \longrightarrow$ Rel min

5D $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + 2\cos\left(\frac{\pi}{6}\right)$

$= \frac{\pi}{6} + 2\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6} + \sqrt{3}$

$f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + 2\cos\left(\frac{5\pi}{6}\right)$

$= \frac{5\pi}{6} + 2\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{5\pi}{6} - \sqrt{3}$

$f\left(\frac{13\pi}{6}\right) = \frac{13\pi}{6} + 2\cos\left(\frac{13\pi}{6}\right)$

$= \frac{13\pi}{6} + 2\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{13\pi}{6} + \sqrt{3}$

$f\left(\frac{17\pi}{6}\right) = \frac{17\pi}{6} + 2\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{17\pi}{6} - \sqrt{3}$

5D

Rel Max at $\left(\frac{\pi}{6}, \frac{\pi}{6} + \sqrt{3}\right)$

$\left(\frac{13\pi}{6}, \frac{13\pi}{6} + \sqrt{3}\right)$

Rel Min at

$\left(\frac{5\pi}{6}, \frac{5\pi}{6} - \sqrt{3}\right)$

$\left(\frac{5\pi}{6}, \frac{5\pi}{6} - \sqrt{3}\right)$

III 5. $f(x) = 3x(x-5)^{2/3}$ on $(-\infty, \infty)$

NOTE: THIS IS ALL YOU NEED TO WRITE

$f'(x) = \frac{5x-15}{\sqrt[3]{x-5}}$

C.V.

TYPE I $f' = 0$

$5x - 15 = 0$

$\Rightarrow x = 3$

TYPE II f' D.N.E

$\sqrt[3]{x-5} = 0$

$\Rightarrow x - 5 = 0$

$\Rightarrow x = 5$

0 4 6

3 5

we have

$f'(0) = \frac{5(0)-15}{\sqrt[3]{0-5}} = \frac{NEG}{NEG} = POS \nearrow$

$f'(4) = \frac{5(4)-15}{\sqrt[3]{4-5}} = \frac{POS}{NEG} = NEG \searrow$

$f'(6) = \frac{5(6)-15}{\sqrt[3]{6-5}} = \frac{POS}{POS} = POS \nearrow$

$\nearrow \quad \searrow \quad \nearrow$

3 5

5D

$f \nearrow : (-\infty, 3) ; (5, \infty)$

$f \searrow : (3, 5)$

CONT.

⑤ Cont: potential extrema at $x=3$ and $x=5$

$f(3) = 3(3)(3-5)^{2/3}$
$= 9(-2)^{2/3}$
≈ 14.3

$f(5) = 3(5)(5-5)^{2/3}$
$= 0$

so

$$\boxed{\begin{array}{l}\text{Rel max at } (3, 14.3) \\ \text{Rel min at } (5, 0)\end{array}}$$

---

⑥ $f(x) = \sin^2(x)$ on $\left(0, \frac{3\pi}{2}\right)$

$f'(x) = 2\sin(x)\cos(x)$

C.V.

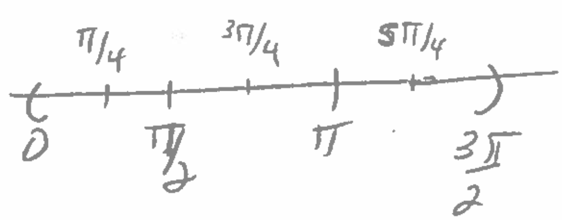
TYPE I $f'=0$

$\Rightarrow 2\sin(x)\cos(x) = 0$

$\Rightarrow \sin(x) = 0$ OR $\cos(x) = 0$

$\Rightarrow \cancel{x=0}$ $\quad x = \pi/2$

$x = \pi$ $\quad \cancel{x = 3\pi/2}$



so TEST VALUES:

$f'(\pi/4) = 2\sin(\pi/4)\cos(\pi/4)$
$= (\text{pos})(\text{pos})(\text{pos})$
$= \text{pos} \nearrow$

Q II

$f'\left(\frac{3\pi}{4}\right) = 2\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{3\pi}{4}\right)$
$= (\text{pos})(\text{pos})(\text{NEG})$
$= \text{NEG} \searrow$

Q III

$f'(5\pi/4) = 2\sin\left(\frac{5\pi}{4}\right)\cos\left(\frac{5\pi}{4}\right)$
$= (\text{pos})(\text{NEG})(\text{NEG})$
$= \text{pos} \nearrow$

so

$$\boxed{\begin{array}{l}f\nearrow: \left(0, \pi/2\right) \;\&\; \left(\pi, 3\pi/2\right) \\ f\searrow: \left(\pi/2, \pi\right)\end{array}}$$

POTENTIAL MAX at $\pi/2$
potential min at π

$f(\pi/2) = \sin^2(\pi/2)$
$= (1)^2$
$= 1$

$f(\pi) = \sin^2(\pi)$
$= (0)^2$
$= 0$

SO $\boxed{\begin{array}{l} \text{Rel max at } (\pi/2, 1) \\ \text{Rel min at } (\pi, 0) \end{array}}$

III

7. $f''(x) = \dfrac{24x^2+32}{(x^2-4)^3}$

C.V.

TYPE I $f''=0$

$\Rightarrow 24x^2+32=0$

$\Rightarrow x^2 = -32/24$

$\Rightarrow x = \pm\sqrt{-32/24}$

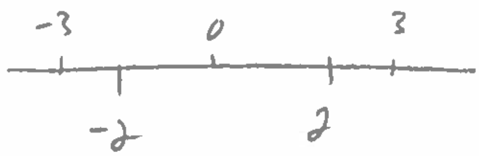
NO SOL'N

TYPE II f'' D.N.E.

$\Rightarrow (x^2-4)^3=0$

$\Rightarrow x^2-4=0$

$\Rightarrow x=2 \quad x=-2$



SO

$f''(-3) = \dfrac{24(-3)^2+32}{((-3)^2-4)^3}$

$= \dfrac{\text{POS}}{\text{POS}}$

$= \text{POS} \Rightarrow \cup$

$f''(0) = \dfrac{24(0)+32}{((0)^2-4)^3}$

$= \dfrac{\text{POS}}{\text{NEG}}$

$= \text{NEG} \Rightarrow \cap$

$f''(3) = \dfrac{24(3)^2+32}{((3)^2-4)^3}$

$= \dfrac{\text{POS}}{\text{POS}}$

$= \text{POS} \Rightarrow \cup$

SO

f is $\cup$: $(-\infty,-2)$; $(2,\infty)$

f is $\cap$: $(-2,2)$

possible inflection pts at

$x=-2 \quad x=2$

$f(-2) = \dfrac{(2)^2}{(2)^2-4} = \dfrac{4}{0}$ DNE $\Rightarrow$

$f(2) = \dfrac{(2)^2}{} = \dfrac{4}{0}$ D.N.E.

8. $\boxed{\text{NO INFLECTION POINTS}}$

VI

9 } same problem
13

FIND $l = x =$
$w = x =$
$h = y =$

MAXIMIZE

$V = x \cdot x \cdot y$
$= x^2 y$

also
$x + x + y = 150$
$\Rightarrow y = 150 - 2x$
if $y = 0$
$\Rightarrow x = 75$

so: $V(x) = x^2(150 - 2x)$
$= 150x^2 - 2x^3$
on $[0, 75]$

so $V'(x) = 300x - 6x^2$

C.V. TYPE I:
$300x - 6x^2 = 0$
$6x(50 - x) = 0$
$\Rightarrow x = 0 \quad x = 50$

TYPE II
NONE
POLYNOMIAL

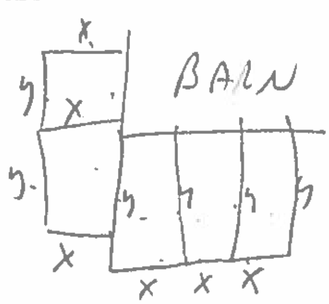
so $V(0) = 150(0)^2 - 2(0)^3 = 0$
$V(50) = 150(50)^2 - 2(50)^3 =$
$= 375000 - 250000$
$= 125000$ — MAX
$V(75) = 150(75)^2 - 2(75)^3$
$= 0$

so $x = 50$
$y = 150 - 2(50)$
$= 150 - 100$
$= 50$

$\Rightarrow \boxed{\begin{array}{l} x = 50 \\ y = 50 \end{array}}$ $\boxed{\begin{array}{l} l = 50 \\ w = 50 \\ h = 50 \end{array}}$

---

(14)



FIND $\boxed{\begin{array}{l} x = \\ y = \end{array}}$

MAXIMIZE
$A = 5xy$
with $6x + 6y = 600$
$\Rightarrow y = 100 - x$
$\Rightarrow A(x) = 5x(100 - x)$
$= 500x - 5x^2$
on $[0, 100]$

$A'(x) = 500 - 10x$

C.V.
TYPE I
$500 - 10x = 0$
$x = 50$

TYPE II
NONE
(POLY)

$A(0) = 0 - 0 = 0$
$A(50) = 500(50) - 5(50)^2$
$= 25000 - 12500$
$= 12500$ — MAX
$A(100) = 500(100) - 5(100)^2$
$= 50000 - 50000 = 0$

so $\boxed{x = 50, \ y = 50}$