① $\sum_{n=1}^{n} \frac{(-1)^n n^2}{n^2+2}$

ALTERNATING SERIES TEST

① $\lim_{n\to\infty} \frac{n^2}{n^2+2} = \lim_{n\to\infty} \frac{n^2}{n^2}$ Dominant term

$= \lim_{n\to\infty} 1$

$= 1 \neq 0$ $\boxed{\therefore \text{ diverges}}$

②. $\sum_{n=1}^{\infty} \frac{5n-3}{n^3+n-6}$

Limit Comparison Test vs $\sum \frac{5n}{n^3} \to \sum \frac{1}{n^2}$ p-series $p=2>1$ converges ←

$\lim_{n\to\infty} \frac{\frac{5n-3}{n^3+n-6}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{5n-3}{n^3+n-6} \cdot \frac{n^2}{1}$

$= \lim_{n\to\infty} \frac{5n^3-3n^2}{n^3+n-6}$

$= \lim_{n\to\infty} \frac{5n^3}{n^3}$ Dominant Term

$= 5$ which is finite and positive

$\boxed{\therefore \text{ converges}}$

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③ $\sum_{n=0}^{\infty} 9\left(\frac{5}{3}\right)^n$ geometric with

$a = 9 \quad r = \frac{5}{3}$

$|r| = \left|\frac{5}{3}\right| = \frac{5}{3} > 1$ $\boxed{\therefore \text{DIVERGES}}$

④ $\sum_{n=1}^{\infty} \frac{(-1)^n 4^{2n}}{n!}$

Ratio test

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} 4^{2(n+1)}}{(n+1)!}}{\frac{(-1)^n 4^{2n}}{n!}} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} 4^{2n+2}}{(n+1)!} \cdot \frac{n!}{(-1)^n 4^{2n}} \right|$$

$$= \lim_{n\to\infty} \frac{4^2}{n+1}$$

$$= \frac{16}{\infty}$$

$$= 0 < 1$$ $\boxed{\therefore \text{Converges ABSOLUTELY}}$

⑤ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n!}{[1\cdot 3\cdot 5 \cdot \ldots \cdot (2n-1)]}$

Ratio test

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+2} (n+1)!}{[1\cdot 3\cdot 5 \cdot \ldots \cdot (2n-1)(2n+1)]}}{\frac{(-1)^{n+1} n!}{[1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)]}} \right| = \lim_{n\to\infty} \left| \frac{(-1)^{n+2} (n+1)!}{[1\cdot 3\cdot 5 \cdot \ldots (2n-1)(2n+1)]} \cdot \frac{[1\cdot 3\cdot 5 \cdot \ldots (2n-1)]}{(-1)^{n+1} n!} \right|$$

$$= \lim_{n\to\infty} \frac{n+1}{2n+1}$$

$$= \lim_{n\to\infty} \frac{n}{2n}$$ Dominant Term

$$= \frac{1}{2} < 1$$ $\boxed{\therefore \text{converges Absolutely}}$

⑥ $\sum_{n=0}^{\infty} 4(-.22)^n$

Geometric $a=4$, $r=-.22$ $\Rightarrow$ $|r| = |-.22| = .22 < 1$ $\boxed{\therefore \text{ converges}}$

$S = \frac{a}{1-r} = \frac{4}{1-.22} = \boxed{\frac{4}{.78}} \approx \boxed{5.128}$

ok

NOT required

⑦ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^2}{n-6}$

alt. series test

① $\lim_{n\to\infty} \frac{n^2}{n-6} = \lim_{n\to\infty} \frac{n^2}{n}$ Dominant Term

$= \lim_{n\to\infty} n$

$= \infty \neq 0$ $\boxed{\therefore \text{ DIVERGES}}$

⑧ $\sum_{n=0}^{\infty} \frac{n \cdot 3^n}{n!}$

Ratio Test $\lim_{n\to\infty} \left| \frac{\frac{(n+1)\,3^{n+1}}{(n+1)!}}{\frac{n\,3^n}{n!}} \right| = \lim_{n\to\infty} \left| \frac{\cancel{(n+1)}\,\cancel{3^{n+1}}^{\,3}}{\cancel{(n+1)!}_{\cancel{(n+1)}}} \cdot \frac{\cancel{n!}}{n\,\cancel{3^n}} \right|$

$= \lim_{n\to\infty} \frac{3}{n}$

$= \frac{3}{\infty}$

$= 0 < 1$ $\boxed{\therefore \text{ converges absolutely}}$

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(9). $\sum_{n=0}^{\infty} \pi (e-2)^n$

Geometric $a = \pi$ $r = (e-2) \approx .718 < 1$

$\boxed{\therefore \text{ CONVERGES}}$

The sum is

$$S = \frac{a}{1-r} = \frac{\pi}{1-(e-2)} = \boxed{\frac{\pi}{3-e}}$$

(10). $\sum_{n=1}^{\infty} \frac{n^k}{n^{k+1}+1}$

Limit comparison vs. $\sum_{n=1}^{\infty} \frac{n^k}{n^{k+1}} = \sum_{n=1}^{\infty} \frac{1}{n}$ (Harmonic Series) p-series $p = 1$ Divergent.

Dominant Terms

$$\lim_{n\to\infty} \frac{\frac{n^k}{n^{k+1}+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^k}{n^{k+1}+1} \cdot \frac{n}{1}$$

$$= \lim_{n\to\infty} \frac{n^{k+1}}{n^{k+1}+1}$$

$$= \lim_{n\to\infty} \frac{n^{k+1}}{n^{k+1}} \quad \text{Dominant Terms}$$

$$= \lim_{n\to\infty} 1$$

$= 1 \rightarrow$ finite & positive

$\therefore$ They agree $\boxed{\therefore \text{ Diverges}}$

(11). $\sum_{n=1}^{\infty} \left(\frac{n}{n+2} - \frac{n+1}{n+3}\right)$ Telescoping?

$\frac{n}{n+2} \rightarrow$ replace "n" with "n+1" $\rightarrow \frac{n+1}{n+1+2} = \frac{n+1}{n+3}$ ✓ yes

(i) $\lim_{n\to\infty} \frac{n+1}{n+3} = \lim_{n\to\infty} \frac{n}{n}$ Dominant terms

$$= \lim_{n\to\infty} 1$$

$= 1$ finite

$\boxed{\therefore \text{ Converges}}$

(ii) The sum is

$$S = \frac{n}{n+2}\Big|_{n=1} - \lim_{n\to\infty} \frac{n+1}{n+3}$$

$$= \frac{1}{3} - 1$$

$$= \boxed{-2/3}$$ ✓