CALC I SOLUTIONS
EXAM #3 SPRING 2019

(1). $f(x) = x^4 - 4x^3 + 10$ on $[-1, 2]$

$f'(x) = 4x^3 - 12x^2$

CRIT. VAL

TYPE I: $f' = 0$

$4x^3 - 12x^2 = 0$

$\Rightarrow 4x^2(x-3) = 0$

$\Rightarrow x = 0$ OR $\cancel{x = 3}$

TYPE II f' D.N.E.

NONE (polynomial)

SO $f(-1) = (-1)^4 - 4(-1)^3 + 10$

$= 1 + 4 + 10 = 15$

$f(0) = (0)^4 - 4(0)^3 + 10$

$= 0 - 0 + 10 = 10$

$f(2) = (2)^4 - 4(2)^3 + 10$

$= 16 - 32 + 10 = -6$

ABS MAX	ABS MIN
$(-1, 15)$	$(2, -6)$

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(2). $f(x) = (x+2)^{2/3} + 4$ on $[-10, -1]$

$f'(x) = \dfrac{2}{3\sqrt[3]{x+2}}$

CRIT. VALS

TYPE I $f' = 0$

$\Rightarrow 2 = 0$

NO SOL'N

TYPE II f' D.N.E.

$\Rightarrow \sqrt[3]{x+2} = 0$

$\Rightarrow x + 2 = 0$

$\Rightarrow x = -2$

SO

$f(-10) = ((-10) + 2)^{2/3} + 4$

$= (-8)^{2/3} + 4$

$= 4 + 4 = 8$

$f(-2) = ((-2) + 2)^{2/3} + 4$

$= 0^{2/3} + 4 = 4$

$f(-1) = ((-1) + 2)^{2/3} + 4$

$= (1)^{2/3} + 4$

$= 1 + 4 = 5$

SO

ABS MAX	ABS MIN
$(-10, 8)$	$(-2, 4)$

II

4. $f(x) = x + 2\cos(x)$ on $(0, \pi)$ || $f''(x) = -2\cos(x)$ pg 2 wo

$f'(x) = 1 - 2\sin(x)$

CRIT. VALS

TYPE I: $f' = 0$

$1 - 2\sin(x) = 0$

$\Rightarrow \sin(x) = 1/2$

QI $x_R = \pi/6$

QII so $x = \pi/6$

$x = 5\pi/6$

$f''(\pi/6) = -2(\cos(\pi/6))$

$= (\text{NEG})(\text{POS})$

$= \text{neg} < 0$

$\Rightarrow$ Rel Max

$f''(5\pi/6) = -2(\cos(5\pi/6))$

$= (\text{neg})(\text{neg})$

$= \text{pos} > 0$

$\Rightarrow$ Rel min

so

Rel max $x = \pi/6$

Rel min $x = 5\pi/6$

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3. $f(x) = x^4 - 4x^3 + 4x^2$

$f'(x) = 4x^3 - 12x^2 + 8x$

CRIT. VALS $f' = 0$

$4x^3 - 12x^2 + 8x = 0$

$\Rightarrow 4x(x^2 - 3x + 2) = 0$

$\Rightarrow 4x(x-2)(x-1) = 0$

$\Rightarrow x = 0 \quad x = 2 \quad x = 1$

TYPE II NONE

Polynomial

$f''(x) = 12x^2 - 24x + 8$

$\Rightarrow f''(0) = 12(0)^2 - 24(0) + 8$

$= 0 - 0 + 8 = 8 > 0$ Rel min

$f''(2) = 12(2)^2 - 24(2) + 8$

$= 48 - 48 + 8 = 8 > 0$ Rel min

$f''(1) = 12(1)^2 - 24(1) + 8$

$= 12 - 24 + 8 = -4 < 0$ Rel max

Rel min at $x = 0$

Rel min at $x = 2$

Rel max at $x = 1$

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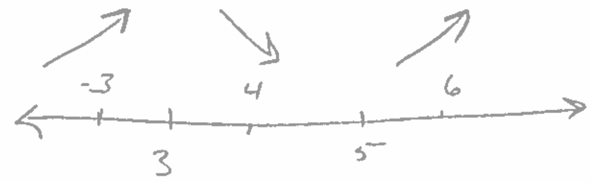
III (5) $f(x) = 9x(x-5)^{2/3}$ on $(-\infty, \infty)$



$f'(x) = \dfrac{15x-45}{\sqrt[3]{x-5}}$

CRIT. VALS

TYPE I $f'=0$

$\Rightarrow 15x-45=0$

$\Rightarrow x=3$

TYPE II f' D.N.E

$\Rightarrow \sqrt[3]{x-5}=0$

$\Rightarrow x-5=0$

$\Rightarrow x=5$

$f'(-3) = \dfrac{15(-3)-45}{\sqrt[3]{-3-5}} = \dfrac{-90}{-2}$ pos > 0 ↗

$f'(4) = \dfrac{15(4)-45}{\sqrt[3]{4-5}} = \dfrac{15}{\sqrt[3]{-1}} = \dfrac{\text{neg}}{\text{pos}} = \text{neg} < 0$ ↘

$f'(6) = \dfrac{15(6)-45}{\sqrt[3]{6-5}} = \dfrac{45}{1}$ $\dfrac{\text{pos}}{\text{pos}} = \text{pos} > 0$ ↗

$\frac{5}{5}$

$\boxed{\begin{array}{l} f \nearrow : (-\infty, 3) \text{ \& } (5, \infty) \\ f \searrow : (3,5) \end{array}}$

$\boxed{\begin{array}{l} \text{Rel max at } x=3 \\ \text{Rel min at } x=5 \end{array}}$

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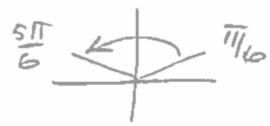
(6) $f(x) = \sin^2(x)$ on $(\pi/6, 5\pi/6)$



$f'(x) = 2\sin(x)\cos(x)$

CRIT. VALS

TYPE I

$2\sin(x)\cos(x)=0$

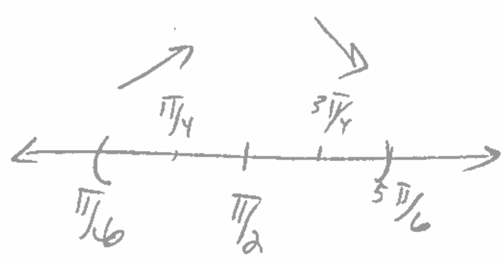
$\Rightarrow \sin(x)=0$ $\quad \cos(x)=0$

$\Rightarrow x = \cancel{0, \pi}$ $\quad x = \underline{\pi/2}$ $\cancel{3\pi/2}$

TYPE II

NONE

(sine cosine)



$f'(\pi/4) = 2\sin(\pi/4)\cos(\pi/4)$

$= (\text{pos})(\text{pos})(\text{pos}) = \text{pos}$ ↗

$f'(5\pi/4) = 2\sin(3\pi/4)\cos(3\pi/4)$

$= (\text{pos})(\text{pos})(\text{neg}) = \text{neg}$ ↘

$\frac{5}{2}$

$\boxed{\begin{array}{l} f \nearrow (\pi/6, \pi/2) \\ f \searrow (\pi/2, 5\pi/6) \end{array}}$

$\boxed{\text{Rel max at } x = \pi/2}$

III $f(x) = \dfrac{x}{x^2-9}$ $\quad f'(x) = -\dfrac{(x^2+9)}{(x^2-9)^2}$ $\quad f''(x) = \dfrac{(2x)(x^2+27)}{(x^2-9)^3}$ pg 4ow

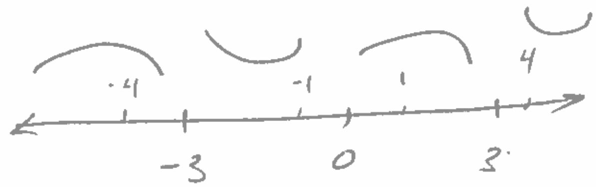
7 Critical Values

TYPE I $f''=0$ TYPE II f'' DNE.

$(2x)(x^2+27)=0$ $\qquad (x^2-9)^3=0$

$\Rightarrow 2x=0 \quad x^2+27=0 \quad \Rightarrow x^2-9=0$

$\Rightarrow x=0 \quad$ no sol'n $\quad \Rightarrow x=3,\,-3$



$f''(-4) = \dfrac{2(-4)\,((-4)^2+27)}{((-4)^2-9)^3}$

$= \dfrac{(\text{NEG})(\text{POS})}{(\text{POS})} = \text{neg}$ ∩

$f''(-1) = \dfrac{2(-1)\,((-1)^2+27)}{((-1)^2-9)^3}$

$= \dfrac{(\text{neg})(\text{pos})}{(\text{neg})} = \text{pos}$ ∪

$f''(1) = \dfrac{2(1)\,((1)^2+27)}{((1)^2-9)^3}$

$= \dfrac{(\text{pos})(\text{pos})}{(\text{neg})} = \text{neg}$ ∩

$f''(4) = \dfrac{2(4)\,((4)^2+27)}{((4)^2-9)^3}$

$= \dfrac{(\text{pos})(\text{pos})}{(\text{pos})} = \text{pos}$ ∪

So

$\boxed{\begin{array}{l} f \text{ is } \cup : (-3,0) \;\&\; (3,\infty) \\ f \text{ is } \cap : (-\infty,-3) \;\&\; (0,3) \end{array}}$

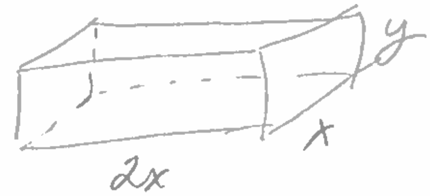
8 $f(-3)$ D.N.E

$f(3)$ DNE.

$f(0) = \dfrac{0}{0-9} = 0$

$\boxed{\text{Inflection point at } (0,0)}$

VI ⑨ Find $\boxed{\begin{aligned} l &= 2x \\ w &= x \\ h &= y \end{aligned}}$



Maximize $V = 2x \cdot x \cdot y$
$= 2x^2 y$

with $2x + x + y = 9$
$\Rightarrow 3x + y = 9$
$\Rightarrow y = 9 - 3x$

$V = 2x^2(9-3x)$
$\Rightarrow V(x) = 18x^2 - 6x^3 \quad 0 \le x \le 3$

so
$V'(x) = 36x - 18x^2$

C.U. TYPE I
$36x - 18x^2 = 0$
$\Rightarrow 18x(2-x) = 0$
$\Rightarrow x = 0 \quad x = 2$

TYPE II
NONE

so
$V(0) = 0 - 0 = 0$
$V(3) = 18(3)^2 - 6(3)^3$
$= 162 - 162 = 0$
$V(2) = 18(2)^2 - 6(2)^3$
$= 72 - 48 = 24$
$\hookrightarrow y = 9 - 3(2)$
$= 3$

MAX Volume is 24
when $\boxed{\begin{aligned} x &= 2 \\ y &= 3 \end{aligned}}$

so $\boxed{\begin{aligned} l &= 4 \\ w &= 2 \\ h &= 3 \end{aligned}}$

Pg 6.x

(10) Maximize $A = 8xy$

Find $\boxed{\begin{matrix} x = \\ y = \end{matrix}}$

with $12x + 6y = 240$

$\Rightarrow 6y = 240 - 12x$

$\Rightarrow y = 40 - 2x$

so $A = 8x(40 - 2x)$

$\Rightarrow A(x) = 320x - 16x^2 \quad 0 \le x \le 20$

$\Rightarrow A'(x) = 320 - 32x$

C.V.

TYPE I

$320 - 32x = 0$

$\Rightarrow x = 10$

TYPE II

NONE

so

$A(0) = 0 - 0 = 0$

$A(20) = 3200 - 3200 = 0$

$A(10) = 160(10) - 8(10)^2$

$= 1600 - 800$

$= 800$

$\hookrightarrow y = 40 - 2(10)$

$= 40 - 20$

$= 20$

MAX AREA IS 800

when $\boxed{\begin{matrix} x = 10 \\ y = 20 \end{matrix}}$