II.

(15) $\int \frac{1}{x+5}\,dx = \int \frac{1}{u}\,du$

Let $u = x+5$
$du = dx$

$= \ln|u| + C$

$= \boxed{\ln|x+5| + C}$

(16) $\int \frac{\ln(x)}{x}\,dx = \int u\,du$

Let $u = \ln(x)$
$du = \frac{1}{x}dx$

$= \frac{u^2}{2} + C$

$= \boxed{\frac{(\ln(x))^2}{2} + C}$

(17) $\int e^x \cos(e^x)\,dx = \int \cos(u)\,du$

Let $u = e^x$
$du = e^x dx$

$= \sin(u) + C$

$= \boxed{\sin(e^x) + C}$

(18) $\int x^2 e^{x^3-4}\,dx = \frac{1}{3}\int e^u\,du$

Let $u = x^3-4$
$du = 3x^2dx$
$\frac{1}{3}du = x^2dx$

$= \frac{1}{3}e^u + C$

$= \boxed{\frac{1}{3}e^{(x^3-4)} + C}$

(19.) $\int \frac{x^2-2}{x^3-6x+1}\,dx = \frac{1}{3}\int \frac{1}{u}\,du$

Let $u = x^3-6x+1$

$\Rightarrow du = (3x^2-6)\,dx$

$\Rightarrow \frac{1}{3}du = (x^2-2)\,dx$

$= \frac{1}{3}\ln|u| + C$

$= \boxed{\frac{1}{3}\ln|x^3-6x+1| + C}$

(20.) $\int \frac{\sec(x)\tan(x)}{\sqrt{1+3\sec(x)}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du$

Let $u = 1+3\sec(x)$

$\Rightarrow du = 3\sec(x)\tan(x)\,dx$

$\Rightarrow \frac{1}{3}du = \sec(x)\tan(x)\,dx$

$= \frac{1}{3}\int u^{-1/2}\,du$

$= \frac{1}{3}\cdot\frac{2}{1}u^{1/2} + C$

$= \boxed{\frac{2}{3}(1+3\sec(x))^{1/2} + C}$

(21.) $\int \frac{8e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int 8e^{(x^{1/2})}(x^{-1/2})\,dx = 16\int e^u\,du$

Let $u = x^{1/2}$

$\Rightarrow du = \frac{1}{2}x^{-1/2}\,dx$

$\Rightarrow 16\,du = 8x^{-1/2}\,dx$

$= 16e^u + C$

$= \boxed{16e^{(x^{1/2})} + C}$

(22.) $\int \frac{e^{5x}}{5+5e^{5x}}\,dx \longrightarrow = \frac{1}{25}\int \frac{1}{u}\,du$

Let $u = 5+5e^{5x}$

$\Rightarrow du = 5e^{5x}(5)\,dx$

$\Rightarrow du = 25e^{5x}\,dx \Rightarrow \frac{1}{25}du = e^{5x}\,dx$

$= \frac{1}{25}\ln|u| + C$

$= \boxed{\frac{1}{25}\ln|5+e^{5x}| + C}$

(23) $\int \frac{4}{\sqrt{81-x^2}}\,dx = 4\int \frac{1}{\sqrt{a^2-u^2}}\,du$

"$\sin^{-1}$ form"

$u = x \qquad a = 9$

$\Rightarrow du = dx$

$\Rightarrow 1du = 1dx$

$= 4\sin^{-1}\left(\frac{u}{a}\right) + C$

$= \boxed{4\sin^{-1}\left(\frac{x}{9}\right) + C}$

---

(24) $\int \frac{8x}{5+x^4}\,dx = \int \frac{8x}{5+(x^2)^2}\,dx = 4\int \frac{1}{a^2+u^2}\,du$

"$\tan^{-1}$ form"

$u = x^2 \rightarrow a = \sqrt{5}$

$\Rightarrow du = 2x\,dx$

$\Rightarrow 4du = 8x\,dx$

$= (4)\left(\frac{1}{a}\right)\tan^{-1}\left(\frac{u}{a}\right) + C$

$= \boxed{\frac{4}{\sqrt{5}}\tan^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + C}$

(25) $\int \frac{3}{5x\sqrt{16x^2-9}}\,dx \Rightarrow (3)\left(\frac{1}{4}\right)\int \frac{1}{\frac{5}{4}u\sqrt{u^2-a^2}}\,du$

$\sec^{-1}$ form

$u = 4x \rightarrow \frac{1}{4}u = x$

$\Rightarrow du = 4\,dx \rightarrow \frac{5}{4}u = 5x$

$\Rightarrow \frac{1}{4}du = dx \qquad a = 3$

$= \frac{\frac{3}{4}}{\frac{5}{4}}\int \frac{1}{u\sqrt{u^2-a^2}}\,du$

$= \frac{3}{5}\left(\frac{1}{a}\right)\sec^{-1}\left(\frac{|u|}{a}\right) + C$

$= \boxed{\frac{3}{5(3)}\sec^{-1}\left(\frac{|4x|}{3}\right) + C}$

pg4wr

(26) $\int \frac{x+2}{4+x^2}\,dx = \int \frac{x}{4+x^2}\,dx + \int \frac{2}{4+x^2}\,dx$

$= \int \frac{x}{4+x^2}\,dx$ $= \frac{1}{2}\int \frac{1}{u}\,du$

Let $u = 4+x^2$
$\Rightarrow du = 2x\,dx$
$\Rightarrow \frac{1}{2}du = x\,dx$

$= \frac{1}{2}\ln|u| + C_1$

$= \boxed{\frac{1}{2}\ln|4+x^2| + C_1}$

part I

$= 2\int \frac{1}{4+x^2}\,dx$

$\tan^{-1}$ form

$u = x$ $a = 2$
$du = dx$

$= 2\int \frac{1}{a^2+u^2}\,du$

$= 2\left(\frac{1}{a}\right)\tan^{-1}\left(\frac{u}{a}\right) + C_2$

$= \frac{2}{2}\tan^{-1}\left(\frac{x}{2}\right) + C_2$

So we have $\boxed{\frac{1}{2}\ln|4+x^2| + \tan^{-1}\left(\frac{x}{2}\right) + C}$

III. (27) $\int_1^e \frac{1}{x(1+\ln(x))}\,dx = \int_1^2 \frac{1}{u}\,du$

Let $u = 1+\ln(x)$
$\Rightarrow du = \frac{1}{x}\,dx$

Limits
upper $u = 1+\ln(e) = 2$
Lower $u = 1+\ln(1) = 1$

$= \left[\ln|u|\right]_1^2$

$= \ln(2) - \ln(1)$ $= 0$

$= \boxed{\ln(2)}$

(28) $\displaystyle\int_0^{\pi/2} 2\sin(x)\, e^{\cos(x)}\, dx =$

pg. Sike

Let $u = \cos(x)$

$\Rightarrow \quad du = -\sin(x)\, dx$

$\Rightarrow \quad -du = \sin(x)\, dx$

$\Rightarrow \quad -2\, du = 2\sin(x)\, dx$

Limits upper $u = \cos(\pi/2) = 0$

lower $u = \cos(0) = 1$

$= -2\displaystyle\int_1^0 e^u\, du$

$= -2\left[e^u\right]_1^0$

$= -2\left[e^0 - e^1\right]$

$= \boxed{-2\left[1 - e\right]}$