SOLUTIONS

6 pts ea

I.

1. $f(x) = \ln(\tan(x)) \Rightarrow f'(x) = \boxed{\frac{1}{\tan(x)} \cdot \sec^2(x)}$

2. $f(x) = x^2 e^{1/\sqrt[3]{x}}$
$= x^2 e^{(x^{-1/3})} \longrightarrow f'(x) = \boxed{x^2 \cdot e^{(x^{-1/3})}\left(-\tfrac{1}{3}x^{-4/3}\right) + e^{(x^{-1/3})}(2x)}$

3. $f(x) = \frac{4x + \ln(4x)}{e^x - 4} \Rightarrow f'(x) = \boxed{\frac{(e^x-4)\left(4 + \frac{1}{4x}(4)\right) - (4x + \ln(4x))(e^x)}{(e^x-4)^2}}$

4. $f(x) = e^{(\cos(x))} \longrightarrow f'(x) = \boxed{e^{\cos(x)}(-\sin(x))}$

5. $f(x) = \ln(\sqrt{3x+x^2}) \Rightarrow f'(x) = \boxed{\frac{1}{\sqrt{3x+x^2}}\left(\tfrac{1}{2}\right)(3x+x^2)^{-\frac{1}{2}}(3+2x)}$

OR

$= \frac{1}{2}\ln(3x+x^2) \xrightarrow{\text{OR}} f'(x) = \boxed{\frac{1}{2}\frac{1}{(3x+x^2)}(3+2x)}$

6. $f(x) = x^3 \ln(x^2+2) \Rightarrow f'(x) = \boxed{x^3\left(\frac{1}{x^2+2}(2x)\right) + \ln(x^2+2)(3x^2)}$

7. $f(x) = \ln(\ln(x)) \Rightarrow f'(x) = \boxed{\frac{1}{\ln(x)}\left(\frac{1}{x}\right)}$

8. $f(x) = e^{(e^x)} \longrightarrow f'(x) = \boxed{e^{(e^x)}(e^x)}$

9. $f(x) = \dfrac{e^{(4x)} + \ln(x^2)}{\sec(x) + x} \Rightarrow$

$$\Rightarrow f'(x) = \boxed{\dfrac{(\sec(x)+x)\left(e^{(4x)}(4) + \frac{1}{x^2}(2x)\right) - \left(e^{(4x)} + \ln(x^2)\right)(\sec(x)\tan(x) + 1)}{(\sec(x)+x)^2}}$$

10. $f(x) = \sec(\ln(x)) \Rightarrow f'(x) = \boxed{\sec(\ln(x))\tan(\ln(x))\left(\frac{1}{x}\right)}$

II

11. $\displaystyle\int \frac{5}{x+1}\,dx = 5\int \frac{1}{x+1}\,dx = 5\int \frac{1}{u}\,du = 5\ln|u| + C$

$= \boxed{5\ln|x+1| + C}$

Let $u = x+1$
$du = dx$

12. $\displaystyle\int \frac{3\ln(x)}{x}\,dx = 3\int \frac{\ln(x)}{x}\,dx = 3\int u\,du = \frac{3u^2}{2} + C$

$= \boxed{\frac{3}{2}(\ln(x))^2 + C}$

Let $u = \ln(x)$
$du = \frac{1}{x}\,dx$

(13). $\displaystyle\int \frac{36e^{6x}}{6+6e^{6x}}\,dx = 36\int \frac{e^{6x}}{6+6e^{6x}} = \int \frac{1}{u}\,du$

Let $u = 6+6e^{6x}$

$\Rightarrow du = 6e^{6x}(6)\,dx$

$\Rightarrow du = 36e^{6x}\,dx$

$= \ln|u| + C$

$= \boxed{\ln|6+6e^{6x}| + C}$

(14) $\displaystyle\int 12x^2 e^{(2x^3-4)}\,dx = 2\int e^u\,du = 2e^u + C$

$= \boxed{2e^{(2x^3-4)} + C}$

Let $u = 2x^3 - 4$

$\Rightarrow du = 6x^2\,dx$

$\Rightarrow 2du = 12x^2\,dx$

(15). $\displaystyle\int (x-3)(x+4)^3\,dx = \int (u-7)u^3\,du$

Let $u = x+4 \Rightarrow u-4 = x$

$du = dx \quad \Rightarrow u-7 = x-3$

DOUBLE LINEAR

$\displaystyle = \int (u^4 - 7u^3)\,du$

$\displaystyle = \frac{u^5}{5} - \frac{7u^4}{4} + C = \boxed{\frac{(x+4)^5}{5} - \frac{7(x+4)^4}{4} + C}$

(16). $\displaystyle\int \frac{8e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int 8e^{(x^{1/2})}(x^{-1/2})\,dx = 16\int e^u\,du$

$= 16e^u + C$

Let $u = x^{1/2}$

$\Rightarrow du = \frac{1}{2}x^{-1/2}\,dx$

$\Rightarrow 16du = 8x^{-1/2}\,dx$

$= \boxed{16\,e^{(x^{1/2})} + C}$