SOLUTIONS

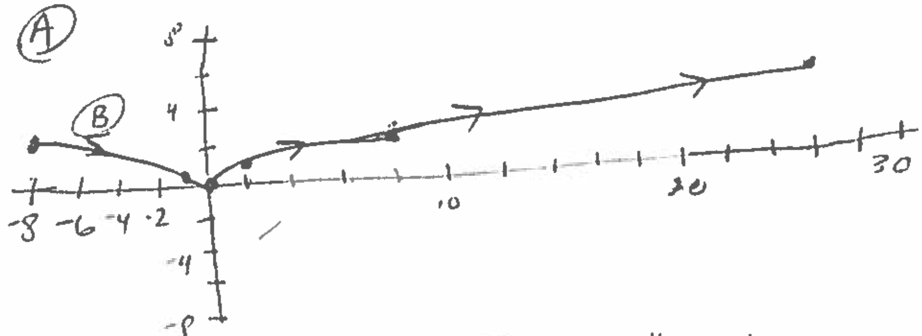
pg 1 n

(I) $x = t^3$, $y = \frac{t^2}{2}$, $-2 \le t \le 3$

(I)

t	x	y	(x, y)
-2	-8	2	(-8, 2)
-1	-1	1/2	(-1, 1/2)
0	0	0	(0, 0)
1	1	1/2	(1, 1/2)
2	8	2	(8, 2)
3	27	9/2	(27, 9/2)

$\Rightarrow$ (A)



(C) $x = t^3 \Rightarrow t = \sqrt[3]{x} = x^{1/3}$

$\Rightarrow y = \frac{(x^{1/3})^2}{2}$

$\Rightarrow \boxed{y = \frac{x^{2/3}}{2}}$

(D) Replace "t" with "-t" and simplify:

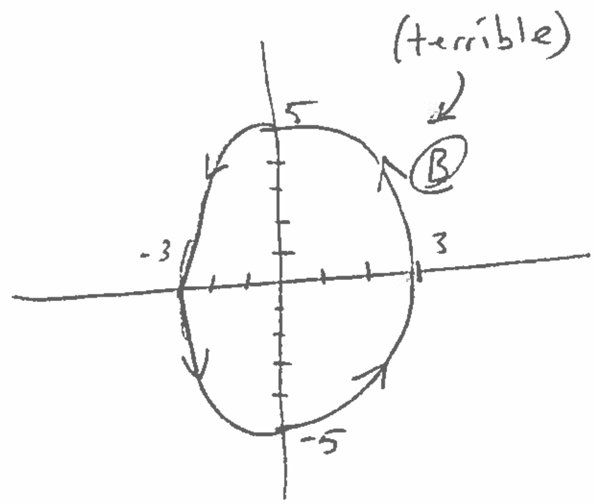
$\Rightarrow \begin{cases} x = (-t)^3 \\ y = \frac{(-t)^2}{2} \end{cases}; \; -2 \le (-t) \le 3 \Rightarrow 2 \ge t \ge -3$

$\Rightarrow \boxed{\begin{aligned} x &= -t^3 \\ y &= \frac{t^2}{2} \end{aligned} \; ; \; -3 \le t \le 2}$

(II) $x = -3\sin(t)$, $y = 5\cos(t)$, $0 \le t \le 2\pi$

t	x	y	(x, y)
0	0	5	(0, 5)
π/2	-3	0	(-3, 0)
π	0	-5	(0, -5)
3π/2	3	0	(3, 0)
2π	0	5	(0, 5)

$\Rightarrow$ (A)



(C) $x = -3\sin(\theta) \Rightarrow \sin(\theta) = \frac{x}{-3}$

$y = 5\cos(\theta) \Rightarrow \cos(\theta) = \frac{y}{5}$

$\Rightarrow \sin^2(\theta) + \cos^2(\theta) = 1$

$\Rightarrow \boxed{\frac{x^2}{9} + \frac{y^2}{25} = 1}$

(D) $\Rightarrow \begin{cases} x = -3\sin(-t) \\ y = 5\cos(-t) \end{cases}; \; 0 \le (-t) \le 2\pi$

$\Rightarrow 0 \ge t \ge -2\pi$

$\Rightarrow -2\pi \le t \le 0$

$\Rightarrow \boxed{\begin{cases} x = 3\sin(t) \\ y = 5\cos(t) \end{cases}; \; 0 \le t \le 2\pi}$

III $\begin{cases} x = t^3 \\ y = \frac{t^2}{2} = \frac{1}{2}t^2 \end{cases}$ $-2 \le t \le 3$.

pg two

(A) $\frac{dy}{dt} = \frac{2(\frac{1}{2})t}{3t^2} = \frac{t}{3t^2} = \frac{1}{3t}$

if $t=2$

$\Rightarrow m = \left.\frac{dy}{dt}\right|_{t=2} = \frac{1}{3(2)} = \frac{1}{6}$

and

$\rightarrow x = (2)^3 = 8$

$y = \frac{(2)^2}{2} = 2$

so, we have

$y - \underline{2} = \underline{1/6}(x - \underline{8})$

$\Rightarrow y - 2 = 1/6\, x - 4/3$

$\Rightarrow \boxed{y = 1/6\, x + 2/3}$

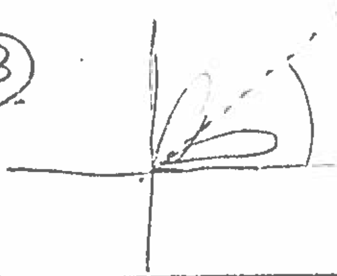
(B) $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{1}{3t}\right]}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left[3t^{-1}\right]}{3t^2} = \frac{-3t^{-2}}{3t^2} = \frac{-1}{t^4}$

so $\left.\frac{-1}{t^4}\right|_{t=-1} = \frac{-1}{(-1)^4} = -1$

$\therefore$ Concave DOWN

IIII (A) $\boxed{A = \left(\frac{1}{2}\right)\int_{-\pi/2}^{\pi/2} (2 - 2\sin(\theta))^2 \, d\theta}$

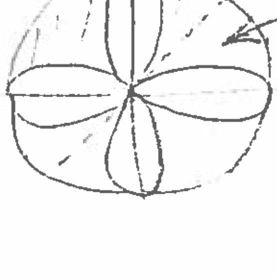
(B)



$\pi/4$ ← By symmetry

so $\boxed{A = \left(\frac{1}{2}\right)\int_0^{\pi/4} (4\sin(4\theta))^2 \, d\theta}$

(C) "outer² minus inner²"



it is enough to find the portion from $0 \le \theta \le \pi/4$ and multiply by 8.

so $\boxed{A = 8\left(\frac{1}{2}\right)\left[\int_0^{\pi/4} (2)^2 \, d\theta - \int_0^{\pi/4} (2\cos(2\theta))^2 \, d\theta\right]}$