(I)

① solve in $(-\infty, \infty)$

$2\cos(x)\tan(x) + \tan(x) = 0$

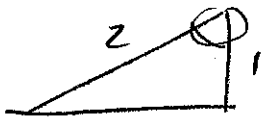
$\Rightarrow \tan(x)(2\cos(x)+1) = 0$

$\Rightarrow \tan(x) = 0$ or $2\cos(x)+1 = 0$

$\Rightarrow x = 0$ or $x = \pi$ (quadrantal) or $\cos(x) = -\frac{1}{2}$

Quadrant Check

QII QIII



$\hookrightarrow x_R = \pi/3$

QII: $\pi/3$, $2\pi/3$; QIII: $\pi/3$, $4\pi/3$

So

$\Rightarrow$

$x = 0 \pm 2\pi n$

$x = \pi \pm 2\pi n$

$x = 2\pi/3 \pm 2\pi n$

$x = 4\pi/3 \pm 2\pi n$

② solve in $(-\infty, \infty)$

$2\sin(x)\cos^2(x) = \sin(x)$

NOTE: DO NOT Divide Both sides by $\sin(x)$!

MOVE TO ONE SIDE OF EQU.

$\Rightarrow 2\sin(x)\cos^2(x) - \sin(x) = 0$

FACTOR

$\Rightarrow \sin(x)(2\cos^2(x) - 1) = 0$

$\Rightarrow \sin(x) = 0$ or $2\cos^2(x) - 1 = 0$

QUADRANTAL

$\Rightarrow x = 0$ or $x = \pi$

$\Rightarrow \cos^2(x) = \frac{1}{2}$

$\Rightarrow \cos(x) = \pm\sqrt{\frac{1}{2}}$

$= \pm\frac{1}{\sqrt{2}}$

QUADRANT CHECK: ALL FOUR (since "$\pm$").

$\hookrightarrow x_R = \pi/4$

QI $\pi/4$; QII $3\pi/4$

QIII $5\pi/4$; QIII $\frac{7\pi}{4}$

So

$x = 0 \pm 2\pi n$

$x = \pi \pm 2\pi n$

$x = \pi/4 \pm 2\pi n$

$x = 3\pi/4 \pm 2\pi n$

$x = 5\pi/4 \pm 2\pi n$

$x = 7\pi/4 \pm 2\pi n$

③. SOLVE IN $[0, 2\pi)$

$3\sin(x)\tan^2(x) = \sin(x)$

AGAIN, DO NOT "divide both sides by $\sin(x)$"

* move to one side:

$3\sin(x)\tan^2(x) - \sin(x) = 0$

* factor

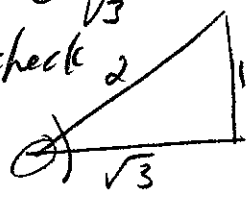
$\Rightarrow \sin(x)(3\tan^2(x) - 1) = 0$

$\sin(x) = 0$ or $x = \pi$ (Quadrantal) or $3\tan^2(x) - 1 = 0$

$\Rightarrow \tan^2(x) = \frac{1}{3}$

$\Rightarrow \tan(x) = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$

Quadrant check All Four.



$\Rightarrow x_R = \pi/6$

QI: $\pi/6$; QII: $5\pi/6$; QIII: $7\pi/6$; QIV: $11\pi/6$

SO $x = \boxed{0, \pi, \pi/6, 5\pi/6, 7\pi/6, 11\pi/6}$

SOLVE IN $[0, 2\pi)$ pg 2 WU

④. $3\sec^2(x) - 4 = 0$

$\Rightarrow \sec^2(x) = \frac{4}{3}$

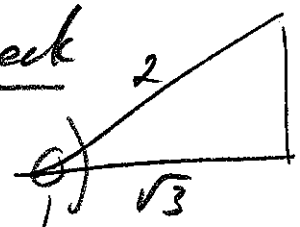
$\Rightarrow \frac{1}{\sec^2(x)} = \frac{1}{4/3}$

$\Rightarrow \cos^2(x) = \frac{3}{4}$

$\Rightarrow \cos(x) = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \cos(x) = \pm\frac{\sqrt{3}}{2}$

Quadrant check all four



$\rightarrow x_R = \pi/6$

QI: $\pi/6$; QII: $5\pi/6$;

QIII: $7\pi/6$ QIII $11\pi/6$

SO $x = \boxed{\pi/6, 5\pi/6, 7\pi/6, 11\pi/6}$

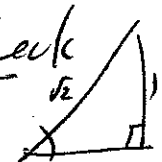
solve in $[0, 2\pi)$

⑤ $\tan(2x) + 1 = 0$

$\Rightarrow \tan(2x) = -1$

Quadrant Check

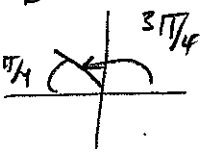
QII QIIII



$\hookrightarrow (2x)_R = \pi/4$

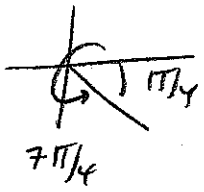
In QII

$(2x)_1 = 3\pi/4$



In QIIII

$(2x) = 7\pi/4$



So

$$\begin{cases} 2x = \frac{3\pi}{4} \pm 2\pi n \\ 2x = \frac{7\pi}{4} \pm 2\pi n \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{3\pi}{8} \pm \pi n \\ x = \frac{7\pi}{8} \pm \pi n \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{3\pi}{8} \pm \frac{8\pi n}{8} \\ x = \frac{7\pi}{8} \pm \frac{8\pi n}{8} \end{cases}$$

Let

	$n=0$	$n=1$	~~$n=2$~~
$x =$	$\frac{3\pi}{8}$	$\frac{11\pi}{8}$	~~$\frac{19\pi}{8}$~~
$x =$	$\frac{7\pi}{8}$	$\frac{15\pi}{8}$	

⑥ solve in $[0, 2\pi)$ pg three

$-2\cos^2(x) - 3\sin(x) + 3 = 0$

MUST GET THE SAME FUNCTION:

So $-2(1 - \sin^2(x)) - 3\sin(x) + 3 = 0$

$\Rightarrow -2 + 2\sin^2(x) - 3\sin(x) + 3 = 0$

$\Rightarrow 2\sin^2(x) - 3\sin(x) + 1 = 0$

FACTOR

$\Rightarrow (2\sin(x) - 1)(\sin(x) - 1) = 0$

$\Rightarrow 2\sin(x) - 1 = 0$ or $\sin(x) - 1 = 0$

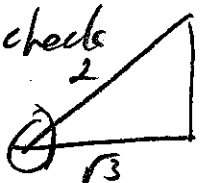
$\Rightarrow \sin(x) = \frac{1}{2}$ or $\sin(x) = 1$

Quadrantal

$x = \pi/2$

Quadrant check

QI QII



$\hookrightarrow x_R = \pi/6$

In QI $\pi/6$

In QII $\hookrightarrow 5\pi/6$

So $\boxed{x = \pi/6, 5\pi/6, \pi/2}$

(7) Solve in $[0, 2\pi)$

$\sqrt{2}\sin(x)\sec(x)+\sec(x)=0$

$\Rightarrow \sec(x)\left(\sqrt{2}\sin(x)+1\right)=0$

$\Rightarrow \sec(x)=0$ OR $\sqrt{2}\sin(x)+1=0$

$\Rightarrow \frac{\text{calc}}{\sec(x)} = \frac{1}{0}$

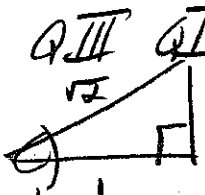
$\Rightarrow \cos(x) = \frac{1}{0}$

undefined

∴ NO SOLUTION

$\Rightarrow \sin(x) = -\frac{1}{\sqrt{2}}$

$\Rightarrow$ Quadrant check



$\rightarrow x_R = \pi/4$

In QIII: $\Rightarrow x = 5\pi/4$

In QIIII: $x = 7\pi/4$

So $x = 5\pi/4 ; 7\pi/4$

Solve in $[0, 2\pi)$

(8) $\cos(3x)+1=0$

$\Rightarrow \cos(3x) = -1$

QUADRANTAL

$\Rightarrow 3x = \pi$ ✓ Base solution

$\Rightarrow 3x = \pi \pm 2\pi n$

$\Rightarrow x = \frac{\pi}{3} \pm \frac{2\pi n}{3}$

Let $n=0$: $x = \pi/3$

$n=1$: $x = 3\pi/3 = \pi$

$n=2$: $x = 5\pi/3$

$n=3$: $x = 7\pi/3$ ~~×~~

SECOND BATCH OF PROBLEMS

① SOLVE IN $[0, 2\pi)$

$$2\cos^2(x) = \cos(x) + 1$$

$$\Rightarrow 2\cos^2(x) - \cos(x) - 1 = 0$$

FACTOR

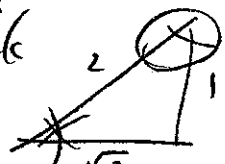
$$\Rightarrow (2\cos(x) + 1)(\cos(x) - 1) = 0$$

$\Rightarrow 2\cos(x) + 1 = 0$ OR $\cos(x) - 1 = 0$

$\Rightarrow \cos(x) = \frac{-1}{2}$ OR $\cos(x) = 1$

Quadrant check

QII QIII



Quadrantal

$x = 0$

QII: $x = \frac{2\pi}{3}$

$\hookrightarrow x_R = \frac{\pi}{3}$

QIII: $x = \frac{4\pi}{3}$

SO $x = 2\pi/3,\ 4\pi/3,\ 0$

② SOLVE IN $[0, 2\pi)$

$$\cos^2(x) - \sin^2(x) = 0$$

$$\Rightarrow \cos^2(x) - (1 - \cos^2(x)) = 0$$

$$\Rightarrow \cos^2(x) - 1 + \cos^2(x) = 0$$

$$\Rightarrow 2\cos^2(x) - 1 = 0$$

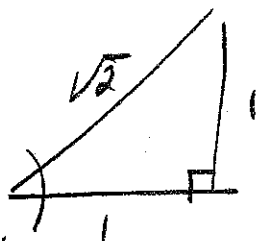
$$\Rightarrow \cos^2(x) = \frac{1}{2}$$

$$\Rightarrow \cos(x) = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{1}}{\sqrt{2}} = \pm\frac{1}{\sqrt{2}}$$

$\Rightarrow \cos(x) = \pm\frac{1}{\sqrt{2}}$

Quadrant Check

ALL FOUR



$\hookrightarrow x_R = \pi/4$

QI: $\pi/4$; QII: $3\pi/4$

QIII: $5\pi/4$; QIV: $7\pi/4$

SO $x = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$

SOLVE IN $[0, 2\pi)$

③ $3\cos(x)\tan^2(x) - \cos(x) = 0$

$\Rightarrow \cos(x)\left(3\tan^2(x) - 1\right) = 0$

$\Rightarrow \cos(x) = 0$ or $3\tan^2(x) - 1 = 0$

$\Rightarrow \tan^2(x) = \frac{1}{3}$

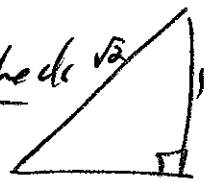
$\Rightarrow \tan(x) = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{1}}{\sqrt{3}}$

$\Rightarrow \tan(x) = \pm\frac{1}{\sqrt{3}}$

Quadrantal

$\Rightarrow x = \pi/2 ; 3\pi/2$

Quadrant check all 4



$\hookrightarrow x_R = \pi/4$

QI $\pi/4$

QII $3\pi/4$

QIII $5\pi/4$ QIIII $7\pi/4$

So $x = \{\pi/2, 3\pi/2, \pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$

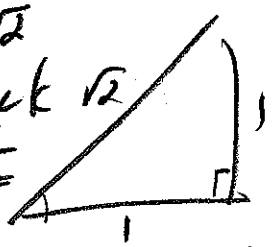
④ Solve in $[0, 2\pi)$

$\sqrt{2}\cos(4x) = -1$

$\Rightarrow \cos(4x) = \frac{-1}{\sqrt{2}}$

Quadrant check

QII QIII



$\hookrightarrow (4x)_R = \pi/4$

QII : $3\pi/4$

QIII : $5\pi/4$

$\Rightarrow 4x = 3\pi/4 \pm 2\pi n$ or $4x = 5\pi/4 \pm 2\pi n$

$\Rightarrow \begin{cases} x = \frac{3\pi}{16} \pm \frac{2\pi n}{4} \\ x = \frac{5\pi}{16} \pm \frac{2\pi n}{4} \end{cases}$

$\Rightarrow \begin{cases} x = \frac{3\pi}{16} \pm \frac{8\pi n}{16} \\ x = \frac{5\pi}{16} \pm \frac{8\pi n}{16} \end{cases}$

Let $n = 0$ $n = 1$ $n = 2$

$x = 3\pi/16$ $x = \frac{11\pi}{16}$ $x = \frac{19\pi}{16}$

$x = 5\pi/16$ $x = \frac{13\pi}{16}$ $x = \frac{21\pi}{16}$

$n = 3$

$x = \frac{27\pi}{16}$

$x = \frac{29\pi}{16}$

$n = 4$

$x = \frac{35\pi}{16}$

⑤ Solve in $[0, 2\pi)$

$$4\sin^2(2x) = 3$$

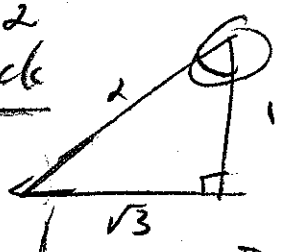
$$\Rightarrow \sin^2(2x) = \frac{3}{4}$$

$$\Rightarrow \sin(2x) = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$$

So: $\sin(2x) = \pm\frac{\sqrt{3}}{2}$

Quadrant Check

● all 4



In QI $\pi/3$

QII $2\pi/3$

QIII $4\pi/3$

QIV $5\pi/3$

$$\Rightarrow \begin{cases} 2x = \pi/3 \pm 2\pi n \\ 2x = 2\pi/3 \pm 2\pi n \\ 2x = 4\pi/3 \pm 2\pi n \\ 2x = 5\pi/3 \pm 2\pi n \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{\pi}{6} \pm \pi n \\ x = \frac{2\pi}{6} \pm \pi n \\ x = \frac{4\pi}{6} \pm \pi n \\ x = \frac{5\pi}{6} \pm \pi n \end{cases}$$

$$\Rightarrow \begin{cases} x = \frac{\pi}{6} \pm \frac{6\pi n}{6} \\ x = \frac{2\pi}{6} \pm \frac{6\pi n}{6} \\ x = \frac{4\pi}{6} \pm \frac{6\pi n}{6} \\ x = \frac{5\pi}{6} \pm \frac{6\pi n}{6} \end{cases}$$

Let $n=0$ $n=1$ $n=2$

$x = \frac{\pi}{6}$	$x = \frac{7\pi}{6}$	~~$x = \frac{13\pi}{6}$~~
$x = \frac{2\pi}{6}$	$x = \frac{8\pi}{6}$	
$x = \frac{4\pi}{6}$	$x = \frac{10\pi}{6}$	
$x = \frac{5\pi}{6}$	$x = \frac{11\pi}{6}$	