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#13 ⓐ $\lim_{x\to\infty} \frac{x^2+2}{x^3-1} = \lim_{x\to\infty} \frac{x^2/x^3 + 2/x^3}{x^3/x^3 - 1/x^3}$

$= \lim_{x\to\infty} \frac{\cancel{1/x}\to 0 + \cancel{2/x^3}\to 0}{1 - \cancel{1/x^3}\to 0}$

$= \frac{0}{1} = \boxed{0}$

ⓑ $\lim_{x\to\infty} \frac{x^2+2}{x^2-1} = \lim_{x\to\infty} \frac{x^2/x^2 + 2/x^2}{x^2/x^2 - 1/x^2}$

$= \lim_{x\to\infty} \frac{1 + \cancel{2/x^2}\to 0}{1 - \cancel{1/x^2}\to 0}$

$= \frac{1}{1} = \boxed{1}$

ⓒ $\lim_{x\to\infty} \frac{x^2+2}{x-1} = \lim_{x\to\infty} \frac{x^2/x + 2/x}{x/x - 1/x}$

$= \lim_{x\to\infty} \frac{x + \cancel{2/x}\to 0}{1 - \cancel{1/x}\to 0}$

$= \frac{\infty}{1} = \boxed{\infty}$

#15 ⓐ $\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x^2-4}$

$= \lim_{x\to\infty} \frac{5/x^2 - 2x^{3/2}/x^2}{3x^2/x^2 - 4/x^2}$

subtract the exponents

$= \lim_{x\to\infty} \frac{\cancel{5/x^2}\to 0 - \cancel{2/x^{1/2}}\to 0}{3 - \cancel{4/x^2}\to 0}$

$= \frac{0}{3} = \boxed{0}$

15 b
$$\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x^{3/2}-4} = \lim_{x\to\infty} \frac{5/x^{3/2} - 2x^{3/2}/x^{3/2}}{3x^{3/2}/x^{3/2} - 4/x^{3/2}}$$

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$$= \lim_{x\to\infty} \frac{\cancel{5/x^{3/2}}\to 0 - 2}{3 - \cancel{4/x^{3/2}}\to 0} = \boxed{\frac{-2}{3}}$$

c
$$\lim_{x\to\infty} \frac{5-2x^{3/2}}{3x-4} = \lim_{x\to\infty} \frac{5/x - 2x^{3/2}/x}{3x/x - 4/x}$$

$$= \lim_{x\to\infty} \frac{\cancel{5/x}\to 0 - 2x^{1/2}}{3 - \cancel{4/x}\to 0}$$

$$= \lim_{x\to\infty} \frac{2x^{1/2}}{3} = \frac{\infty}{3} = \boxed{\infty}$$

17.
$$\lim_{x\to\infty}\left(4 + \cancel{3/x}\right)\to 0 = 4+0 = \boxed{4}$$

19.
$$\lim_{x\to\infty} \frac{7x+6}{9x-4} = \lim_{x\to\infty} \frac{7x/x + 6/x}{9x/x - 4/x} = \lim_{x\to\infty} \frac{7 + \cancel{6/x}\to 0}{9 - \cancel{4/x}\to 0} = \boxed{\frac{7}{9}}$$

21.
$$\lim_{x\to(-\infty)} \frac{2x^2+x}{6x^3+2x^2+x} = \lim_{x\to(-\infty)} \frac{2x^2/x^3 + x/x^3}{6x^3/x^3 + 2x^2/x^3 + x/x^3}$$

$$= \lim_{x\to\infty} \frac{2 + \cancel{1/x}\to 0}{6 + \cancel{2/x + 1/x^2}\to 0} = \frac{2}{6} = \boxed{\frac{1}{3}}$$

23.
$$\lim_{x\to(-\infty)} \frac{5x^2}{x+3} = \lim_{x\to(-\infty)} \frac{5x^2/x}{x/x + 3/x} = \lim_{x\to(-\infty)} \frac{5x}{1 + \cancel{3/x}\to 0}$$

$$= \frac{5(-\infty)}{1} = \boxed{-\infty}$$

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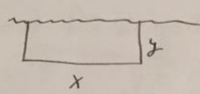
#19



given Area = $405000\,m^2$

MINIMIZE Amount of Fence

Find dimensions $\boxed{\begin{array}{l} x = \quad m \\ y = \quad m \end{array}}$ (meters)

Need a function for amount of fence $\Rightarrow$ $L = x + 2y$

must eliminate a variable

$\Rightarrow$ use Area $= xy = 405000$

$$\Rightarrow y = \frac{405000}{x}$$

so

$$L(x) = x + 2\left(\frac{405000}{x}\right)$$

$$= x + \frac{810000}{x}$$

$$= x + 810000x^{-1}$$

minimum value for $x = 0$

If $y = 0$

$\Rightarrow 0 = \frac{405000}{x}$

$\Rightarrow$ there is no max for x

$\Rightarrow$ interval is $[0, \infty)$

---

so minimize

$L(x) = x + 810000x^{-1}$ on $[0, \infty)$

$$\Rightarrow L'(x) = 1 - 810000x^{-2}$$

$$= 1 - \frac{810000}{x^2}$$

Critical values

TYPE I set $L' = 0$

$$1 - \frac{810000}{x^2} = 0$$

$$\Rightarrow \frac{1}{1} = \frac{810000}{x^2}$$

cross multiply

$\Rightarrow x^2 = 810000$

$\Rightarrow x = \pm 900 \Rightarrow x = 900$

choose positive

TYPE II L' D.N.E.

$\Rightarrow x = 0$

$$L(900) = 900 + \frac{810000}{900}$$

$$= 900 + 900$$

$$= 1800$$

$L(0) = 0 + \frac{810000}{0}$ D.N.E.

so

$x = 900$

$$y = \frac{405000}{900}$$

$$= 450$$

so $\boxed{\begin{array}{l} x = 900\ m \\ y = 450\ m \end{array}}$

(21) Find Dimensions

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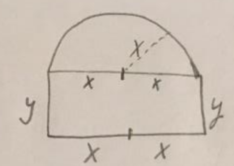
Find $\boxed{x = \quad y = }$



NOTE: IT IS EASIER IF x IS HALF THE HORIZONTAL

Maximize AREA

Given that Perimeter = 16

$$\text{Area} = \underbrace{\left(\begin{smallmatrix}\text{Semi}\\ \text{Circle}\end{smallmatrix}\right)} + (\text{Rectangle})$$

$$= \overbrace{\left(\tfrac{1}{2}\right)(\pi)(x^2)} + (2x)(y)$$

$$= \frac{\pi}{2}x^2 + 2xy$$

MUST ELIMINATE y

Perimeter = 16

$$P = \left(\begin{smallmatrix}\text{Semi}\\ \text{Circle}\end{smallmatrix}\right) + y + 2x + y = 16$$

$$\Rightarrow \pi x + 2x + 2y = 16$$

$$\Rightarrow y = \frac{16 - \pi x - 2x}{2} \quad (**)$$

$$= 8 - \frac{\pi}{2}x - x$$

$\Rightarrow$ min value for $x = 0$

$\Rightarrow$ max value if $y = 0$

$$\Rightarrow \pi x + 2x + 2(0) = 16$$

$$\Rightarrow x(\pi + 2) = 16 \Rightarrow x = \frac{16}{\pi + 2} \approx 3.11$$

$[0, 3.11]$

SO we have

$$A(x) = \frac{\pi}{2}x^2 + 2x\left(8 - \frac{\pi}{2}x - x\right)$$

$$= \frac{\pi}{2}x^2 + 16x - \pi x^2 - 2x^2$$

$$= 16x - \pi/2\, x^2 - 2x^2 \quad (*)$$

SO $A'(x) = 16 - \pi x - 4x$

critical values

TYPE I $A' = 0$

TYPE II NONE (POLYNOMIAL)

$$\Rightarrow 16 - \pi x - 4x = 0$$

$$\Rightarrow 16 = \pi x + 4x$$

$$\Rightarrow 16 = x(\pi + 4)$$

$$\Rightarrow x = \frac{16}{\pi + 4} \approx 2.24$$

SO

$(*)$ $A(2.24) = 16(2.24) - \frac{\pi}{2}(2.24)^2 - 2(2.24)^2 \approx 17.9$

$A(0) = 16(0) - \pi/2\,(0)^2 - 2(0)^2 = 0$

$A(3.11) = 16(3.11) - \pi/2\,(3.11)^2 - 2(3.11)^2 = 15.22$

SO max occurs when $x = 2.24$

$(**)$ $y = \dfrac{16 - \pi(2.24) - 2(2.24)}{2} \approx 2.24$

$$\boxed{\begin{array}{l} x = 2.24\text{ ft} \\ y = 2.24\text{ ft} \end{array}}$$