① for $f(x) = \sec(x) + \beta x^2$
What value or values must β assume if f is concave up at $x = \pi$?

$\Rightarrow$ concave up $\Rightarrow f'' > 0$

$$\Rightarrow f'(x) = \sec(x)\tan(x) + 2\beta x$$

$$\Rightarrow f''(x) = (\sec(x))(\sec^2(x)) + (\tan(x))(\sec(x)\tan(x)) + 2\beta$$

$$= \sec^3(x) + \sec(x)\tan^2(x) + 2\beta$$

concave up when $x = \pi \Rightarrow$

$$\Rightarrow \sec^3(\pi) + \sec(\pi)\tan^2(\pi) + 2\beta > 0$$

$$\Rightarrow \frac{1}{\cos^3(\pi)} + 0 + 2\beta > 0$$

$$\Rightarrow \frac{1}{(-1)^3} + 2\beta > 0$$

$$\Rightarrow -1 + 2\beta > 0$$

$$\Rightarrow \boxed{\beta > \tfrac{1}{2}}$$

② $f(x) = 6x^3 - 6\beta x^2 + 5$. What value or values of β will cause f to be decreasing at $x=$

"f is decreasing at $x=2$" means "$f' < 0$ at $x=2$"

$$\Rightarrow f'(x) = 18x^2 - 12\beta x < 0 \text{ at } x = 2$$

$$\Rightarrow 18(2)^2 - 12\beta(2) < 0$$

$$\Rightarrow 18(4) - 24\beta < 0$$

$$\Rightarrow 72 - 24\beta < 0$$

$$\Rightarrow -24\beta < -72$$

$$\Rightarrow \beta > \frac{-72}{-24}$$

$$\Rightarrow \beta > 3$$

③ What value of β will cause $\lim\limits_{x\to\infty} \frac{x^3+5}{x^{2\beta}+1} = 1$?

we have

$$\lim_{x\to\infty} \frac{x^3+5}{x^{2\beta}+1} = \lim_{x\to\infty} \frac{\frac{x^3}{x^{2\beta}} + \frac{5}{x^{2\beta}}}{\frac{x^{2\beta}}{x^{2\beta}} + \frac{1}{x^{2\beta}}} \quad \text{assuming } \beta > 0$$

$$= \lim_{x\to\infty} \frac{\frac{x^3}{x^{2\beta}} + 0}{1 + 0}$$

$$= \lim_{x\to\infty} \frac{x^3}{x^{2\beta}}$$

now, either $2\beta > 3$, $2\beta < 3$, or $2\beta = 3$

Ⓘ If $2\beta > 3$
$\Rightarrow \lim\limits_{x\to\infty} \frac{x^3}{x^{2\beta}} = 0$
Not what we want

Ⓘⓘ If $2\beta < 3$
$\Rightarrow \lim\limits_{x\to\infty} \frac{x^3}{x^{2\beta}} = \infty$
Not what we want

Ⓘⓘⓘ If $2\beta = 3$
$\Rightarrow \lim\limits_{x\to\infty} \frac{x^3}{x^{2\beta}} = 1$
This is what we want

so $2\beta = 3$

$$\Rightarrow \boxed{\beta = \tfrac{3}{2}}$$

④ $f(x) = \sin(x) - \beta x^2$ has an inflection point when $x = \pi/6$. What is β?

"Inflection pt" $\Rightarrow f''(x) = 0$

$$\Rightarrow f'(x) = \cos(x) - 2\beta x$$

$$\Rightarrow f''(x) = -\sin(x) - 2\beta$$

at $x = \pi/6$

$$f''(\pi/6) = -\sin(\pi/6) - 2\beta = 0$$

$$\Rightarrow -\left(\tfrac{1}{2}\right) - 2\beta = 0$$

$$\Rightarrow -2\beta = \tfrac{1}{2}$$

$$\Rightarrow \boxed{\beta = -\tfrac{1}{4}}$$