Extra Delta Epsilon Proofs Homework Assignment

SOLUTIONS

pg 1ne

① $\lim_{x \to 4} (3x+1) = 13$

Ⓘ Let $\epsilon > 0$

Ⓘ Search for $\delta$

We want:

$|f(x) - L| < \epsilon$

$|(3x+1) - 13| < \epsilon$

$|3x - 12| < \epsilon$

$|3(x-4)| < \epsilon$

$|3||x-4| < \epsilon$

$|x-4| < \frac{\epsilon}{|3|}$

Choose $\delta = \frac{\epsilon}{|3|}$

Ⅲ Verify this $\delta$:

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-4| < \frac{\epsilon}{|3|}$

$\Rightarrow |3||x-4| < \epsilon$

$\Rightarrow |3(x-4)| < \epsilon$

$\Rightarrow |3x-12| < \epsilon$

$\Rightarrow |3x+1-1-12| < \epsilon$

$\Rightarrow |(3x+1)-13| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$

② $\lim_{x \to (1)} (5x-7) = -2$

Ⅰ Let $\epsilon > 0$

Ⅱ Search for $\delta$

We want:

$|f(x) - L| < \epsilon$

$|(5x-7) - (-2)| < \epsilon$

$|5x - 5| < \epsilon$

$|5(x-1)| < \epsilon$

$|5||x-1| < \epsilon$

$|x-1| < \frac{\epsilon}{|5|}$

Choose $\delta = \frac{\epsilon}{|5|}$

Ⅲ Verify this $\delta$:

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-1| < \frac{\epsilon}{|5|}$

$\Rightarrow |5||x-1| < \epsilon$

$\Rightarrow |5x-5| < \epsilon$

$\Rightarrow |5x-7+7-5| < \epsilon$

$\Rightarrow |(5x-7)+2| < \epsilon$

$\Rightarrow |(5x-7)-(-2)| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$

③ $\lim_{x \to (-3)} (-2x+4) = 10$

Ⅰ Let $\epsilon > 0$

Ⅱ Search for $\delta$

We want:

$|f(x) - L| < \epsilon$

$|(-2x+4) - 10| < \epsilon$

$|-2x - 6| < \epsilon$

$|-2(x+3)| < \epsilon$

$|-2||x-(-3)| < \epsilon$

$|x-(-3)| < \frac{\epsilon}{|-2|}$

Choose $\delta = \epsilon/|-2|$

Ⅲ Verify this $\delta$:

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-(-3)| < \frac{\epsilon}{|-2|}$

$\Rightarrow |-2||x+3| < \epsilon$

$\Rightarrow |-2x-6| < \epsilon$

$\Rightarrow |-2x+4-4-6| < \epsilon$

$\Rightarrow |(-2x+4)-10| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$

pg2dw

④. $\lim_{x\to 0}(7x-6)=-6$

I Let $\epsilon>0$

II Search for $\delta$:

We want:

$|f(x)-L|<\epsilon$

$|(7x-6)-(-6)|<\epsilon$

$|7x-6+6|<\epsilon$

$|7x|<\epsilon$

$|7||x|<\epsilon$

✱ NOTE: when $a=0$ this is the technique:

$|7||x-0|<\epsilon$ we have obtained "$|x-a|$"

$|x-0|<\frac{\epsilon}{|7|}$

Choose $\delta=\frac{\epsilon}{|7|}$

III Verify this $\delta$:

$0<|x-a|<\delta \Rightarrow$

$\Rightarrow |x-0|<\frac{\epsilon}{|7|}$

$\Rightarrow |7||x|<\epsilon$

$\Rightarrow |7x|<\epsilon$

$\Rightarrow |7x-6+6|<\epsilon$

$\Rightarrow |(7x-6)-(-6)|<\epsilon$

$\Rightarrow |f(x)-L|<\epsilon$ ✓

⑤ $\lim_{x\to 6}(\frac{1}{3}x-5)=-3$

I Let $\epsilon>0$

II Search for $\delta$:

We want:

$|f(x)-L|<\epsilon$

$|(\frac{1}{3}x-5)-(-3)|<\epsilon$

$|\frac{1}{3}x-2|<\epsilon$

$|\frac{1}{3}(x-6)|<\epsilon$

$|\frac{1}{3}||x-6|<\epsilon$

$|x-6|<\frac{\epsilon}{|1/3|}$

If we like we may write this as $|3|\epsilon$

Choose $\delta=\frac{\epsilon}{|1/3|}$

III Verify this $\delta$:

$0<|x-a|<\delta \Rightarrow$

$\Rightarrow |x-6|<\frac{\epsilon}{|1/3|}$

$\Rightarrow |1/3||x-6|<\epsilon$

$\Rightarrow |1/3\,x-2|<\epsilon$

$\Rightarrow |1/3\,x-5+5-2|<\epsilon$

$\Rightarrow |(1/3\,x-5)+3|<\epsilon$

$\Rightarrow |(1/3\,x-5)-(-3)|<\epsilon$

$\Rightarrow |f(x)-L|<\epsilon$ ✓

⑥ $\lim_{x\to 1/4}(8x+1)=3$

I Let $\epsilon>0$

II Search for $\delta$:

We want:

$|f(x)-L|<\epsilon$

$|(8x+1)-3|<\epsilon$

$|8x-2|<\epsilon$

$|8(x-1/4)|<\epsilon$

$|8||x-1/4|<\epsilon$

$|x-1/4|<\frac{\epsilon}{|8|}$

Choose $\delta=\frac{\epsilon}{|8|}$

III Verify this $\delta$:

$0<|x-a|<\delta \Rightarrow \frac{\epsilon}{}$

$\Rightarrow |x-1/4|<\frac{\epsilon}{|8|}$

$\Rightarrow |8||x-\frac{1}{4}|<\epsilon$

$\Rightarrow |8x-2|<\epsilon$

$\Rightarrow |8x+1-1-2|<\epsilon$

$\Rightarrow |(8x+1)-3|<\epsilon$

$\Rightarrow |f(x)-L|<\epsilon$ ✓

pg three

7. $\lim_{x\to(-4)} (-2x-13) = -5$

I Let $\epsilon > 0$

II Search for $\delta$:

We want:

$|f(x) - L| < \epsilon$

$|(-2x-13) - (-5)| < \epsilon$

$|-2x-8| < \epsilon$

$|-2(x+4)| < \epsilon$

$|-2|\,|x-(-4)| < \epsilon$

$|x-(-4)| < \frac{\epsilon}{|-2|}$

Choose $\delta = \frac{\epsilon}{|-2|}$

III Verify this $\delta$:

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-(-4)| < \frac{\epsilon}{|-2|}$

$\Rightarrow |-2|\,|x+4| < \epsilon$

$\Rightarrow |-2x-8| < \epsilon$

$\Rightarrow |-2x-13+13-8| < \epsilon$

$\Rightarrow |(-2x-13)+5| < \epsilon$

$\Rightarrow |(-2x-13)-(-5)| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$

8. $\lim_{x\to(-5)} (-x+2) = 7$

I Let $\epsilon > 0$

II Search for $\delta$:

We want:

$|f(x) - L| < \epsilon$

$|(-x+2) - 7| < \epsilon$

$|-x-5| < \epsilon$

$|-1(x+5)| < \epsilon$

$|-1|\,|x-(-5)| < \epsilon$

$|x-(-5)| < \frac{\epsilon}{|-1|}$

choose $\delta = \frac{\epsilon}{|-1|}$

III Verify this $\delta$:

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-(-5)| < \frac{\epsilon}{|-1|}$

$\Rightarrow |-1|\,|x+5| < \epsilon$

$\Rightarrow |-x-5| < \epsilon$

$\Rightarrow |-x+2-2-5| < \epsilon$

$\Rightarrow |(-x+2) - 7| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$

9. $\lim_{x\to a} (mx) = (ma)$

I Let $\epsilon > 0$

II Search for $\delta$:

We want:

$|f(x) - L| < \epsilon$

$|(mx) - (ma)| < \epsilon$

$|mx - ma| < \epsilon$

$|m(x-a)| < \epsilon$

$|m|\,|x-a| < \epsilon$

$|x-a| < \frac{\epsilon}{|m|}$

choose $\delta = \frac{\epsilon}{|m|}$

III Verify this $\delta$:

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-a| < \frac{\epsilon}{|m|}$

$\Rightarrow |m|\,|x-a| < \epsilon$

$\Rightarrow |mx - ma| < \epsilon$

$\Rightarrow |(mx) - (ma)| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$