Homework Solutions:
Delta Epsilon Proofs Assignment I

Solutions

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① $\lim_{x\to 3}(2x+7)=13$

Ⅰ Let $\epsilon>0$

Ⅱ Search for δ

We want

$|f(x)-L|<\epsilon$

$\to |(2x+7)-13|<\epsilon$

$\to |2x-6|<\epsilon$

$\Rightarrow |2(x-3)|<\epsilon$

$\to |2||x-3|<\epsilon$

$\Rightarrow |x-3|<\frac{\epsilon}{|2|}$

Choose $\delta=\frac{\epsilon}{|2|}$

Ⅲ Verify this δ

$0<|x-a|<\delta \Rightarrow$

$\to |x-3|<\frac{\epsilon}{|2|}$

$\Rightarrow |2||x-3|<\epsilon$

$\to |2(x-3)|<\epsilon$

$\to |2x-6|<\epsilon$

$\Rightarrow |2x+7-7-6|<\epsilon$

$\Rightarrow |(2x+7)-13|<\epsilon$

$\Rightarrow |f(x)-L|<\epsilon$ ✓

② $\lim_{x\to -4}(3x+10)=-2$

Ⅰ Let $\epsilon>0$

Ⅱ Search for δ

We want

$|f(x)-L|<\epsilon$

$\to |(3x+10)-(-2)|<\epsilon$

$\to |3x+12|<\epsilon$

$\to |3(x+4)|<\epsilon$

$\to |3||x+4|<\epsilon$

$\to |x-(-4)|<\frac{\epsilon}{|3|}$

Choose $\delta=\frac{\epsilon}{|3|}$

Ⅲ Verify this δ

$0<|x-a|<\delta \Rightarrow$

$\to |x-(-4)|<\frac{\epsilon}{|3|}$

$\to |3||x-(-4)|<\epsilon$

$\to |3x+12|<\epsilon$

$\to |3x+10-10+12|<\epsilon$

$\to |(3x+10)+2|<\epsilon$

$\Rightarrow |(3x+10)-(-2)|<\epsilon$

$\to |f(x)-L|<\epsilon$ ✓

③ $\lim_{x\to 4}(-5x+14)=-6$

Ⅰ Let $\epsilon>0$

Ⅱ Search for δ

We want

$|f(x)-L|<\epsilon$

$\to |(-5x+14)-(-6)|<\epsilon$

$\to |-5x+20|<\epsilon$

$\to |-5(x-4)|<\epsilon$

$\to |-5||x-4|<\epsilon$

$\Rightarrow |x-4|<\frac{\epsilon}{|-5|}$

Choose $\delta=\frac{\epsilon}{|-5|}$

Ⅲ Verify this δ

$0<|x-a|<\delta \Rightarrow$

$\Rightarrow |x-4|<\frac{\epsilon}{|-5|}$

$\Rightarrow |-5||x-4|<\epsilon$

$\Rightarrow |-5x+20|<\epsilon$

$\Rightarrow |-5x+14-14+20|<\epsilon$

$\Rightarrow |(-5x+14)+6|<\epsilon$

$\Rightarrow |(-5x+14)-(-6)|<\epsilon$

$\Rightarrow |f(x)-L|<\epsilon$ ✓

④ $\lim_{x\to(-1)} (-7x-7) = 0$

I Let $\epsilon > 0$

II Search for δ

We want

$|f(x) - L| < \epsilon$

$|(-7x-7) - 0| < \epsilon$

$|-7x-7| < \epsilon$

$|(-7)(x+1)| < \epsilon$

$|-7||x-(-1)| < \epsilon$

• $|x-(-1)| < \frac{\epsilon}{|-7|}$

Choose $\delta = \frac{\epsilon}{|-7|}$

III. Verify this δ

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-(-1)| < \frac{\epsilon}{|-7|}$

$\Rightarrow |-7||x-(-1)| < \epsilon$

$\Rightarrow |-7x-7| < \epsilon$

$\Rightarrow |(-7x-7)-0| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$ ✓

⑤ $\lim_{x\to(-2)} (-4x-7) = 1$

I Let $\epsilon > 0$

II Search for δ

We want

$|f(x) - L| < \epsilon$

$|(-4x-7) - 1| < \epsilon$

$|-4x-8| < \epsilon$

$|-4(x+2)| < \epsilon$

$|-4||x-(-2)| < \epsilon$

$|x-(-2)| < \frac{\epsilon}{|-4|}$

Choose $\delta = \epsilon/|-4|$

III Verify this δ

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-(-2)| < \frac{\epsilon}{|-4|}$

$\Rightarrow |-4||x+2| < \epsilon$

$\Rightarrow |-4x-8| < \epsilon$

$\Rightarrow |-4x-7+7-8| < \epsilon$

$\Rightarrow |(-4x-7)-1| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$ ✓

⑥ $\lim_{x\to a} (mx+b) = (ma+b)$ pg 200

I Let $\epsilon > 0$

II Search for δ

We want

$|f(x) - L| < \epsilon$

$|(mx+b) - (ma+b)| < \epsilon$

$|mx+b-ma-b| < \epsilon$

$|mx-ma+b-b| < \epsilon$

$|m(x-a)| < \epsilon$

$|m||x-a| < \epsilon$

$|x-a| < \frac{\epsilon}{|m|}$

Choose $\delta = \frac{\epsilon}{|m|}$

III Verify this δ:

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-a| < \frac{\epsilon}{|m|}$

$\Rightarrow |m||x-a| < \epsilon$

$\Rightarrow |m(x-a)| < \epsilon$

$\Rightarrow |mx-ma| < \epsilon$

$\Rightarrow |mx+b-b-ma| < \epsilon$

$\Rightarrow |(mx+b)-(ma+b)| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$ ✓