Pg 84

#39 $f(x) = \frac{6}{x}$ $\boxed{\text{discontinuity at } x=0}$

#40 $f(x) = \frac{4}{x-6}$ $x-6=0 \Rightarrow x=6$

$\boxed{\text{Discontinuity at } x=6}$

#41 $f(x) = \frac{1}{4-x^2}$ $4-x^2=0 \Rightarrow (2+x)(2-x)=0 \rightarrow x=-2 \quad x=2$

$\boxed{\text{Discontinuities at } x=-2, x=2}$

#42 $f(x) = \frac{1}{x^2+1}$ $x^2+1=0 \Rightarrow$ NO SOLUTION

$\rightarrow \boxed{\text{No discontinuities}}$

#43 $f(x) = 3x - \cos(x)$

$\hookrightarrow f(x)$ is continuous everywhere

$\Rightarrow \boxed{\text{No discontinuities}}$

#44 $f(x) = \sin(x) - 8x$

$\hookrightarrow f(x)$ is continuous everywhere

$\hookrightarrow \boxed{\text{No discontinuities}}$

#45 $f(x) = \frac{x}{x^2-x}$

$= \frac{x}{x(x-1)} \rightarrow$ simplifying $\Rightarrow \frac{1}{(x-1)} \Rightarrow$ Indicates: $x=0$ is a removable discontinuity

$\boxed{\text{discontinuity at } x=0 \text{ and } x=1}$

$\rightarrow$ Since the factor "x" cancels, the discontinuity at $x=0$ is removable

#46. $f(x) = \frac{x}{x^2-4} \Rightarrow x^2-4=0 \rightarrow (x-2)(x+2)=0$

$\Rightarrow x=2;\ x=-2$

$\Rightarrow$ $\boxed{\text{Discontinuities at } x=2;\ x=-2}$

#47. $f(x) = \frac{(x+2)}{x^2-3x-10}$

$= \frac{(x+2)}{(x-5)(x+2)}$ $\rightarrow$ $\boxed{\text{since the } (x+2) \text{ factor cancel, the discontinuity at } x=-2 \text{ is Removable}}$

$= \frac{1}{(x-5)}$ $\rightarrow$ $\boxed{\text{Discontinuities at } x=-2 \text{ and } x=5}$

#48. $f(x) = \frac{(x+2)}{x^2-x-6}$

$= \frac{(x+2)}{(x-3)(x+2)}$ $\rightarrow$ $\boxed{\text{Discontinuities at } x=3 \text{ \& } x=-2}$

$= \frac{1}{(x-3)}$ $\rightarrow$ $\boxed{\text{Since the "}(x+2)\text{" factors cancel the discontinuity at } x=-2 \text{ is Removeable}}$

#49. $f(x) = \frac{|x+7|}{(x+7)}$

$$\Rightarrow f(x) = \begin{cases} \frac{x+7}{x+7} & \text{if } (x+7) \geq 0 \Rightarrow \text{if } x \geq -7 \\ \frac{-(x+7)}{x+7} & \text{if } (x+7) < 0 \rightarrow \text{if } x < -7 \end{cases}$$

Cont.

#49 continued

simplifying:

$$f(x) = \begin{cases} 1 & \text{if } x \geq -7 \\ -1 & \text{if } x < -7 \end{cases}$$

Must investigate continuity at $x=-7$

① $f(-7) \overset{TOP}{=} 1$

② $\lim_{x\to(-7)} f(x)$

ⓐ $\lim_{x\to(-7)^-} f(x) \overset{BOT}{=} \lim_{x\to(-7)^-} (-1) = -1$

ⓑ $\lim_{x\to(-7)^+} f(x) \overset{TOP}{=} \lim_{x\to(-7)^+} (1) = 1$

Since the left- and right-hand limits do not agree, $\lim_{x\to(-7)} f(x)$ D.N.E.

So $\boxed{\text{Discontinuity at } x=-7}$

#50 $f(x) = \dfrac{2|x-3|}{(x-3)}$

$$\Rightarrow f(x) = \begin{cases} \dfrac{2(x-3)}{(x-3)} & \text{if } (x-3) \geq 0 \rightarrow x \geq 3 \\ \dfrac{2(-(x-3))}{(x-3)} & \text{if } (x-3) < 0 \Rightarrow x < 3 \end{cases}$$

⇓ CONT.

#50 continued

simplifying:

$$\Rightarrow f(x) = \begin{cases} 2 & \text{if } x \geq 3 \\ -2 & \text{if } x < 3 \end{cases}$$

Must investigate continuity at $x=3$

(i) $f(3) \stackrel{TOP}{=} 2$ ✓

(ii) $\lim_{x \to 3} f(x)$

(a) $\lim_{x \to 3^-} f(x) \stackrel{BOT}{=} \lim_{x \to 3^-} (-2) = -2$ ✓

(b) $\lim_{x \to 3^+} f(x) \stackrel{TOP}{=} \lim_{x \to 3^+} (2) = 2$ ✓

Since the left- and right-hand limits do not agree, $\lim_{x \to 3} f(x)$ D.N.E.

So $\boxed{\text{Discontinuity at } x=3}$

#57

$$f(x) = \begin{cases} \frac{1}{2}x + 1 & \text{if } x \leq 2 \\ x^2 - 4x + 1 & \text{if } x > 2 \end{cases}$$

Must check continuity at $x=2$

(i) $f(2) \stackrel{TOP}{=} \frac{1}{2}(2) + 1 = 2$ ✓

continued

#51 continued

(iii) $\lim_{x \to 2} f(x)$

(a) $\lim_{x \to 2^-} f(x) \overset{\text{TOP}}{=} \lim_{x \to 2^-} \left(\frac{1}{2}x + 1\right) = \frac{1}{2}(2) + 1 = 2$

(b) $\lim_{x \to 2^+} f(x) \overset{\text{BOT}}{=} \lim_{x \to 2^-} (x^2 - 4x + 1) = (2)^2 - 4(2) + 1$
$= 4 - 8 + 1 = -3$

since the left- and right-hand limits do not agree, $\lim_{x \to 2} f(x)$ D.N.E.

so $\boxed{\text{Discontinuity at } x = 2}$

#52.

$$f(x) = \begin{cases} \tan\left(\frac{\pi x}{4}\right) & \text{if } |x| < 1 \\ x & \text{if } |x| \geq 1 \end{cases}$$

that is:

$$f(x) = \begin{cases} \tan\left(\frac{\pi x}{4}\right) & \text{if } -1 < x < 1 \\ x & \text{if } x \geq 1 \text{ or } x \leq -1 \end{cases}$$

Must check continuity at $x = -1$ and $x = 1$

(A) $x = -1$ (i) $f(-1) \overset{\text{BOT}}{=} -1$

(ii) $\lim_{x \to (-1)}$

NOTE: It might be better to write $f(x)$ this way:

$$f(x) = \begin{cases} x & \text{if } x \leq -1 \\ \tan\left(\frac{\pi x}{4}\right) & \text{if } -1 < x < 1 \\ x & \text{if } x \geq 1 \end{cases}$$

(#52) cont. NOTE: I will use this version of the function: pg. 6 of x

$$f(x) = \begin{cases} x & \text{if } x \le -1 \\ \tan\left(\frac{\pi x}{4}\right) & \text{if } -1 < x < 1 \\ x & \text{if } x \ge 1 \end{cases}$$

(a) $\lim_{x \to (-1)^-} f(x) \overset{\text{TOP}}{=} \lim_{x \to (-1)^-} (x) = -1$ ✓

(b) $\lim_{x \to (-1)^+} f(x) \overset{\text{MID}}{=} \lim_{x \to (-1)^+} \tan\left(\frac{\pi x}{4}\right)$

$= \tan\left(\frac{\pi(-1)}{4}\right)$

$= \tan\left(-\frac{\pi}{4}\right)$

$= -1$ ✓

so $\lim_{x \to -1} f(x) = -1$ ✓

(c) since $\lim_{x \to (-1)} f(x) = f(-1)$ ✓

$\therefore f(x)$ is continuous at $x = -1$ ✓

an exactly similar argument shows $f(x)$ is continuous at $x = 1$ ✓

$\boxed{\therefore f(x) \text{ has NO discontinuities}}$

#59. $f(x) = \begin{cases} 3x^2 & x \geq 1 \\ ax-4 & x < 1 \end{cases}$ pg 7 even $\frac{?}{2}$

to be continuous we require

$$\lim_{x \to 1^-} f(x) = \lim_{x \to (1)^+} f(x)$$

that is: BOT TOP

$$\lim_{x \to 1^-} (ax-4) = \lim_{x \to 1^+} (3x^2)$$

$\Rightarrow a(1) - 4 = 3(1)^2$

$\Rightarrow a - 4 = 3 \Rightarrow \boxed{a = 7}$

#60. $f(x) = \begin{cases} 3x^3 & \text{if } x \leq 1 \\ ax+5 & \text{if } x > 1 \end{cases}$

we require $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

that is: BOT TOP

$$\lim_{x \to 1^-} (3x^3) = \lim_{x \to 1^+} (ax+5)$$

$\Rightarrow 3(1)^3 = a(1) + 5$

$\Rightarrow 3 = a + 5$

$\Rightarrow \boxed{a = -2}$

#61 Pg 8ight

$$f(x) = \begin{cases} x^3 & \text{if } x \le 2 \\ ax^2 & \text{if } x > 2 \end{cases}$$

we require $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x)$

that is: $\overset{\text{TOP}}{\lim\limits_{x\to 2^-}} (x^3) = \overset{\text{BOT}}{\lim\limits_{x\to 2^+}} (ax^2)$

$\Rightarrow \quad 2^3 = a(2)^2$

$\Rightarrow \quad 8 = a(4)$

$\Rightarrow \quad \boxed{a = 2}$