Homework Sec 3.4 Pg 1ne

Pg 196 # 33, 35, 37 See ALSO #43 !!

#33 $f(x) = 6x - x^2$

$f'(x) = 6 - 2x$

Critical values

TYPE I: $f' = 0$ TYPE II f' D.N.E.

$\Rightarrow 6 - 2x = 0$ NONE

$\Rightarrow x = 3$ POLYNOMIAL

$f''(x) = -2$

$f''(3) = -2$

"SECOND DERIVATIVE TEST"
EVALUATE f'' at the TYPE I CRITICAL VALUES OBTAINED FROM f'

$f''(3) = -2 < 0 \Rightarrow$ Rel max

finally compute

$f(3) = 6(3) - (3)^2$

$= 18 - 9 = 9$

so $\boxed{\text{Rel Max at } (3, 9)}$

#35 $f(x) = x^3 - 3x^2 + 3$

$f'(x) = 3x^2 - 6x$

Critical Values

TYPE I $f' = 0$ TYPE II $= f'$ D.N.E.

$3x^2 - 6x = 0$ NONE (polynomial)

$3x(x-2) = 0$

$x = 0$ $x = 2$

$f''(x) = 6x - 6$

so

$f''(0) = 6(0) - 6 = -6 < 0$ Rel max

$f''(2) = 6(2) - 6 = 6 > 0$ Rel min

$f(0) = (0)^3 - 3(0) + 3$

$= 0 - 0 + 3$

$= 3$

$f(2) = (2)^3 - 3(2)^2 + 3$

$= 8 - 12 + 3$

$= -1$

so $\boxed{\begin{array}{l}\text{Rel max at } (0, 3) \\ \text{Rel min at } (2, -1)\end{array}}$

#37 $f(x) = x^4 - 4x^3 + 2$

$f'(x) = 4x^3 - 12x^2$

Critical Values

TYPE I: $f' = 0$

$\Rightarrow 4x^3 - 12x^2 = 0$

$\Rightarrow 4x^2(x-3) = 0$

$x = 0 \quad x = 3$

TYPE II

f'' D.N.E.

NONE (polynomial)

$f''(x) = 12x^2 - 24x$

$f''(0) = 12(0)^2 - 24(0)$

$= 0 - 0$

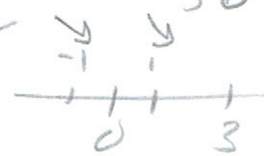
$= 0 \rightarrow$ NO INFO

$f''(3) = 12(3)^2 - 24(3)$

$= 108 - 72$

$= 36 > 0 \rightarrow$ Rel min

go back & do 1st Derivative Analysis on Either side of $x = 0$



$f'(-1) = 4(-1)^3 - 12(-1)^2$

$= -4 - 12 = -16 < 0 \searrow$

$f'(1) = 4(1)^3 - 12(1)^2$

$= 4 - 12 = -8 < 0 \searrow$

Since there is NO CHANGE FROM INCREASING TO DECREASING OR VICE-VERSA, THERE IS NO RELATIVE EXTREMUM AT $x = 0 \Rightarrow$ IT IS A "SADDLE POINT"

so

$f(0) = (0)^4 - 4(0)^3 + 2 = 2$

$f(3) = (3)^4 - 4(3)^3 + 2$

$= 81 - 108 + 2 = -25$

so $\boxed{\text{Rel min at } (3, -25)}$

#43

$f(x) = \cos(x) - x$ on $[0, 4\pi]$

$f'(x) = -\sin(x) - 1$

$f''(x) = -\cos(x)$

Critical Values:

TYPE I: $f' = 0$

$-\sin(x) - 1 = 0$

$\Rightarrow \sin(x) = -1$

$\Rightarrow x = 3\pi/2 \quad x = \frac{7\pi}{2}$

TYPE II f' D.N.E.

NONE SINCE f' is sine and polynomial

SD

$f''\left(\frac{3\pi}{2}\right) = -\cos\left(\frac{3\pi}{2}\right)$

$= 0$

NO INFO

$f''\left(\frac{7\pi}{2}\right) = -\cos\left(\frac{7\pi}{2}\right)$

$= 0$

NO INFO

When the 2nd deriv. test gives no info, must do 1st Derivative analysis.

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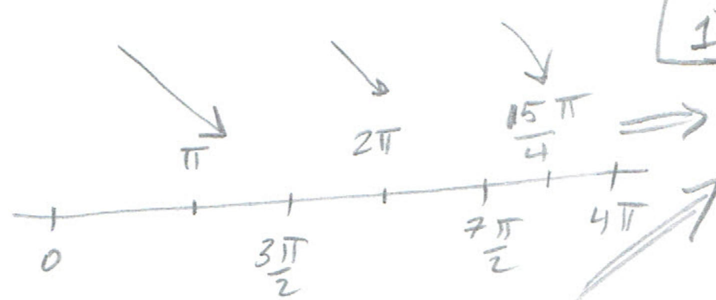
use 1st Derivative:



$f'(\pi) = -\sin(\pi) - 1$

$= -1 < 0$ ↘

$f'(2\pi) = -\sin(2\pi) - 1$

$= 0 - 1$

$= -1 < 0$ ↘

$f'\left(\frac{15\pi}{4}\right) = -\sin\left(\frac{15\pi}{4}\right) - 1$

$= -\left(-\frac{\sqrt{2}}{2}\right) - 1$

$= \frac{\sqrt{2}}{2} - 1$

$\approx .7 - 1 = -.3 < 0$ ↘

since the function never changes from increasing to decreasing (or vice-versa, there in no Relative extremum.