Homework solutions Sec 3.4

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#5. $f(x) = x^2 - 4x + 8$

$\Rightarrow f'(x) = 2x - 4$

$\Rightarrow f''(x) = 2$

Critical values

TYPE I: $f'' = 0$

$\Rightarrow 2 = 0$

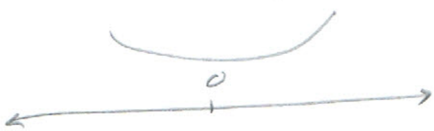
$\Rightarrow$ NO variable, so NO SOL'N

TYPE II: f'' D.N.E.

None since $f''(x)$ is a polynomial

So there are no critical values

Thus there is only 1 region: the entire number line.



Select a representative (any value will do) I select $x=0$

$\Rightarrow f''(0) = 2 > 0$ so f is $\cup$

$\boxed{\text{Thus } f \text{ is } \cup \text{ on } (-\infty, \infty)}$

$\boxed{\text{There are no inflection points}}$ since the concavity never changes.

oops. Just realized we are not asked to find inflection points

#7. $f(x) = x^4 - 3x^3$

$\Rightarrow f'(x) = 4x^3 - 9x^2$

$\Rightarrow f''(x) = 12x^2 - 18x$

Critical values

TYPE I: $f'' = 0$

$\Rightarrow 12x^2 - 18x = 0$

$\Rightarrow 6x(2x-3) = 0$

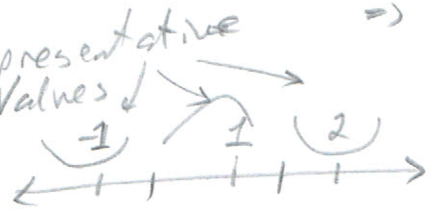
$\Rightarrow x = 0$ or $x = 3/2 = 1.5$

TYPE II f'' D.N.E.

None (polynomial)

Representative Values

so



$f''(-1) = 12(-1)^2 - 18(-1)$
$= 12 + 18 = 30 > 0 \ \cup$

$f''(1) = 12(1)^2 - 18(1)$
$= 12 - 18 = -6 < 0 \ \cap$

$f''(2) = 12(2)^2 - 18(2)$
$= 48 - 36$
$= 12 > 0 \ \cup$

$\boxed{\begin{array}{l} \text{So } f \text{ is } \cup \text{ on } (-\infty, 0) \ \& \ (1.5, \infty) \\ f \text{ is } \cap \text{ on } (0, 1.5) \end{array}}$

#9 $f(x) = \dfrac{24}{x^2+12}$

$$\Rightarrow f'(x) = \frac{(x^2+12)(0) - (24)(2x)}{(x^2+12)^2}$$

$$= \frac{-48x}{(x^2+12)^2}$$

NOTE: MY SOLUTION IN TUESDAY'S ZOOM MEETING CONTAINED AN ERROR. THIS SOLUTION IS CORRECT

$$f''(x) = \frac{(x^2+12)^2(-48) - (-48)(2)(x^2+12)(2x)}{(x^2+12)^4}$$

$$= \frac{\cancel{(x^2+12)}(-48)\left[(x^2+12) - (2)(2x)\right]}{(x^2+12)^4} = \frac{(-48)(-3x^2+12)}{(x^2+12)^3}$$

Critical values:

TYPE I: $f''=0$

$\Rightarrow (-48)(-3x^2+12) = 0$

$\Rightarrow -3x^2 = -12$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

TYPE II: $f''(x)$ D.N.E.

$\Rightarrow (x^2+12)^3 = 0$

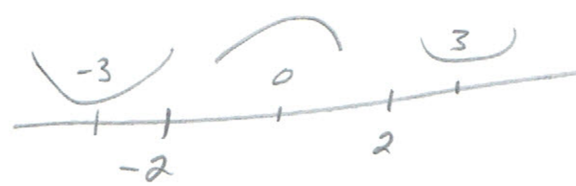
$\Rightarrow x^2+12 = 0$

$\Rightarrow x^2 = -12$

$\Rightarrow x = \pm\sqrt{-12}$

NO SOL'N

SO



$$f''(-3) = \frac{(-48)(-3(-3)^2+12)}{((-3)^2+12)^3} = \frac{(-48)(-15)}{(21)^3} = \frac{720}{9261} > 0 \quad \cup$$

$$f''(0) = \frac{(-48)(-3(0)^2+12)}{((0)^2+12)^3} = \frac{(-48)(12)}{(12)^3} = \frac{-576}{1728} < 0 \quad \cap$$

$$f''(3) = \frac{(-48)(-3(3)^2+12)}{((3)^2+12)^3} = \frac{(-48)(-15)}{(21)^3} = \frac{720}{9261} > 0 \quad \cup$$

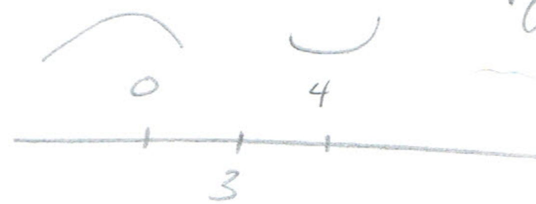
SO

$$\boxed{\begin{array}{l} f \text{ is } \cup \text{ on } (-\infty,-2) \ \& \ (2,\infty) \\ f \text{ is } \cap \text{ on } (-2,2) \end{array}}$$

#17. $f(x) = x^3 - 9x^2 + 24x - 18$

$f'(x) = 3x^2 - 18x + 24$

$f''(x) = 6x - 18$



Critical Values

TYPE I $f'' = 0$ TYPE II f'' D.N.E.

$\Rightarrow 6x - 18 = 0$ NONE (polynomial)

$\Rightarrow x = 3$

$f''(0) = 6(0) - 18 = -18 < 0$ $\cap$

$f''(4) = 6(4) - 18 = 6 > 0$ $\cup$

Potential Inflection point at $x = 3$

NOTE: f'' Here!

NOTE: ORIGINAL FUNCTION HERE!!

$f(3) = (3)^3 - 9(3)^2 + 24(3) - 18$

$= 27 - 81 + 72 - 18$

$= 0$

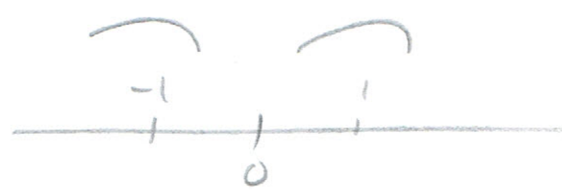
So
$\boxed{\begin{array}{l} f \text{ is } \cup \text{ on } (3, \infty) \\ f \text{ is } \cap \text{ on } (-\infty, 3) \\ \text{Inflection pt at } (3, 0) \end{array}}$

#19

$f(x) = 2 - 7x^4$

$f'(x) = -28x^3$

$f''(x) = -84x^2$



Critical Values

TYPE I: $f'' = 0$ TYPE II f'' D.N.E.

$-84x^2 = 0$ NONE (polynomial)

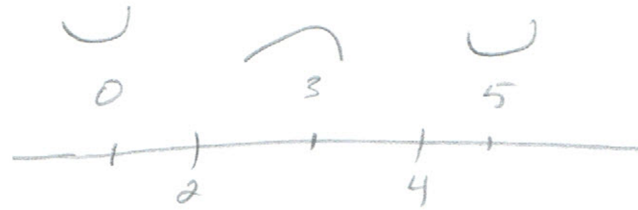
$\Rightarrow x = 0$

$f''(-1) = -84(-1)^2 = -84 < 0$ $\cap$

$f''(1) = -84(1)^2 = -84 < 0$ $\cap$

$\boxed{\begin{array}{l} \text{So } f \text{ is } \cap \text{ on } (-\infty, 0) \text{ \& } (0, \infty) \\ \text{No inflection point since the} \\ \text{concavity never changes.} \end{array}}$

#21 $f(x) = x(x-4)^3$



$f'(x) = x(3)(x-4)^2(1) + (x-4)^3(1)$
$= (x-4)^2(3x + (x-4))$
$= (x-4)^2(4x-4)$

$f''(x) = (x-4)^2(4) + (4x-4)(2)(x-4)(1)$
$= (x-4)((4)(x-4) + (2)(4x-4))$
$= (x-4)(4x-16+8x-8)$
$= (x-4)(12x-24)$

Critical Values

TYPE I: $f''=0$ TYPE II f'' D.N.E.

$(x-4)(12x-24)=0$ NONE (polynomial)

$\Rightarrow x=4 \quad x=2$

$f''(0) = ((0)-4)(12(0)-24)$
$= (-4)(-24) = 96 > 0$ ∪

$f''(3) = (3-4)(12(3)-24)$
$= (-1)(12) = -12 <$ ∩

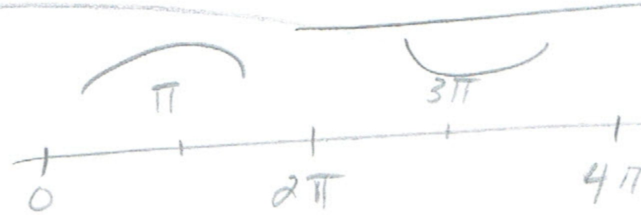
$f''(5) = (5-4)(12(5)-24)$
$= (1)(36) = 36 > 0$ ∪

$f(2) = (2)((2)-4)^3 = (2)(-2)^3 = -16$

$f(4) = (4)(4-4)^3 = 0$

so f is ∪ on $(-\infty, 2)$ & $(4, \infty)$
f is ∩ on $(2,4)$
inflection pts at $(2,-16)$ & $(4,0)$

#27 $f(x) = \sin(\frac{1}{2}x)$ on $[0, 4\pi]$



$f'(x) = \cos(\frac{1}{2}x)(\frac{1}{2}) = \frac{1}{2}\cos(\frac{1}{2}x)$

$f''(x) = -(\frac{1}{2})\sin(\frac{1}{2}x)(\frac{1}{2}) = -\frac{1}{4}\sin(\frac{1}{2}x)$

Critical Values:

TYPE I: $f''=0$ TYPE II = f'' D.N.E.

$-\frac{1}{4}\sin(\frac{1}{2}x)=0$ NONE it is a function of sine.

$\Rightarrow \sin(\frac{1}{2}x)=0$

$\Rightarrow (\frac{1}{2}x) = 0, \pi, 2\pi, 3\pi, 4\pi$

$\Rightarrow x = 0, 2\pi, 4\pi, \cancel{6\pi}, \cancel{8\pi}$

$f''(\pi) = -\frac{1}{4}\sin(\frac{1}{2}(\pi))$
$= -\frac{1}{4}(1) = -\frac{1}{4} < 0$ ∩

$f''(3\pi) = -\frac{1}{4}\sin(\frac{1}{2}(3\pi))$
$= -\frac{1}{4}\sin(\frac{3\pi}{2}) = -\frac{1}{4}(-1)$
$= \frac{1}{4} > 0$ ∪

$f(2\pi) = \sin(\frac{1}{2}(2\pi)) = 0$

so f is ∪ on $(2\pi, 4\pi]$
f is ∩ on $[0, 2\pi)$
Inflection pt at $(2\pi, 0)$