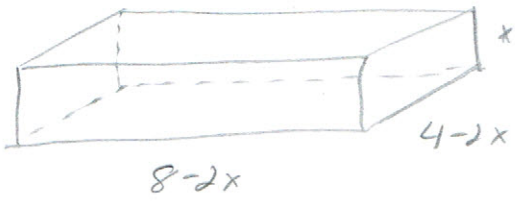
SOLUTIONS:

① 

Find: $x =$ ____ in

$V(x) = (8-2x)(4-2x)x$

$= 32x - 24x^2 + 4x^3$

$\Rightarrow V(x) = 4x^3 - 24x^2 + 32x$

$\Rightarrow V'(x) = 12x^2 - 48x + 32$

$= 4(3x^2 - 12x + 8)$

Interval: $[0, 2]$

Critical values

TYPE I: $V' = 0$

TYPE II - V' D.N.E.
NONE (polynomial)

$4(3x^2 - 12x + 8) = 0$

$\Rightarrow x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(8)}}{2(3)}$

$= \frac{12 \pm \sqrt{48}}{6}$

$\approx \frac{12 \pm 6.9}{6}$ → $x \approx 3.15$ NOT IN $[0,2]$

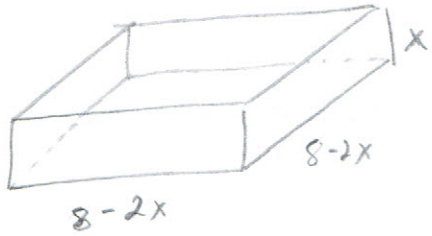
OR → $x \approx .84$

So $V(0) = 4(0)^3 - 24(0)^2 + 32(0) = 0$

$V(2) = 4(2)^3 - 24(2)^2 + 32(2) = 0$

$V(.84) = 4(.84)^3 - 24(.84)^2 + 32(.84) \approx 12.32$ ← MAX

So $\boxed{x = .84 \text{ in}}$

② 

Find $\boxed{V = \quad \text{cm}^3}$

$V(x) = (8-2x)(8-2x)x$

$= 64x - 32x^2 + 4x^3$

$\Rightarrow V(x) = 4x^3 - 32x^2 + 64x$ on $[0, 4]$

$\Rightarrow V'(x) = 12x^2 - 64x + 64$

$= 4(3x^2 - 16x + 16)$

So $V(0) = 4(0)^3 - 32(0)^2 + 64(0) = 0$

$V(4) = 4(4)^3 - 32(4)^2 + 64(4) = 0$

$V(1.3) = 4(1.3)^3 - 32(1.3)^2 + 64(1.3) = 37.9$

Critical Values

TYPE I $V' = 0$

$4(3x^2 - 16x + 16) = 0$

$\Rightarrow x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(3)(16)}}{2(3)}$

$= \frac{16 \pm \sqrt{64}}{6}$

$= \frac{16 \pm 8}{6}$ → $\frac{24}{6} = 4$

OR → $\frac{8}{6} \approx 1.3$

So

Max Volume is 37.9 cm^3

③. $A = 4xy$ $\Rightarrow$ so $A(x) = 4x(-\frac{1}{2}x + 12)$

$= -2x^2 + 48x$ on $[0, 24]$

And $3x + 6y = 72$

$\Rightarrow 6y = -3x + 72$

$\Rightarrow y = -\frac{3}{6}x + \frac{72}{6}$

$= -\frac{1}{2}x + 12$

$\Rightarrow A'(x) = -4x + 48$

critical values

TYPE I $A' = 0$

$\Rightarrow -4x + 48 = 0$

$\Rightarrow x = 12$

TYPE II A' D.N.E.

NONE polynomial

FIND: $A =$ ____ m^2

so

$A(0) = -2(0)^2 + 48(0) = 0$

$A(24) = -2(24)^2 + 48(24) = 0$

$A(12) = -2(12)^2 + 48(12) = 288$ ←

so

MAX Area is

$A = 288\ m^2$

④ THERE ARE 5 pens which are each $x \cdot y$ so the total area is $A = 5xy$

FIND:

MAX

$A =$ ____ mm^2

$x =$ ____ m

$y =$ ____ m

also: $6x + 6y = 60$

$\Rightarrow 6y = -6x + 60$

$\Rightarrow y = -x + 10$

so $A = 5x(-x + 10)$

$\Rightarrow A(x) = -5x^2 + 50x$ on $[0, 10]$

$\Rightarrow A'(x) = -10x + 50$

critical values:

TYPE I: $A' = 0$

$-10x + 50 = 0$

$\Rightarrow x = 5$

TYPE II A' D.N.E.

NONE

POLYNOMIAL

so

$A(0) = -5(0)^2 + 50(0) = 0$

$A(10) = -5(10)^2 + 50(10) = 0$

$A(5) = -5(5)^2 + 50(5) = 125$ ←

↳ $x = 5$ so $y = -(5) + 10$

$= 5$

so MAX Area

$A = 125\ m^2$

$x = 5\ m$

$y = 5\ m$