

#27 [Corrections to Sec 4.4 HW]

$$\int_0^4 |x^2 - 9| dx \Rightarrow \text{cusp at } x=3 \Rightarrow$$

$x^2 - 9 = 0 \Rightarrow x=3 \text{ or } x=-3$
 $x=-3$ not in the interval $[0,4]$ determined by the limits of integration.

$$\Rightarrow \int_0^3 |x^2 - 9| dx + \int_3^4 |x^2 - 9| dx$$

$$= \int_0^3 -(x^2 - 9) dx + \int_3^4 (x^2 - 9) dx$$

$$= \int_0^3 (-x^2 + 9) dx + \int_3^4 (x^2 - 9) dx$$

$$= \left[-\frac{x^3}{3} + 9x \right]_0^3 + \left[\frac{x^3}{3} - 9x \right]_3^4$$

$$= \left[\left(-\frac{(3)^3}{3} + 9(3) \right) - \left(-\frac{(0)^3}{3} + 9(0) \right) \right] + \left[\left(\frac{(4)^3}{3} - 9(4) \right) - \left(\frac{(3)^3}{3} - 9(3) \right) \right]$$

$$= \left[(-9 + 27) - (0 + 0) \right] + \left[\left(\frac{64}{3} - 36 \right) - (9 - 27) \right]$$

$$= [18] + \left[\left(\frac{64}{3} - 36 \right) - (-18) \right] = 18 + \frac{64}{3} - 36 + 18 = \boxed{\frac{64}{3}}$$

#47 (REGARDING THE EXAMS, YOU MAY IGNORE #47 & #49. BUT FOR THE RECORD, HERE ARE THE SOLUTIONS.)

#47 The M.V.T. for integrals says there is c in $[a,b]$ such that $\int_a^b f(x) dx = f(c)(b-a)$

so for $f(x) = x^3$ on $[0,3]$

$$\int_0^3 x^3 dx = \left[\frac{x^4}{4} \right]_0^3$$

$$= \frac{81}{4}$$

$$f(c) = c^3 \text{ and } (b-a) = (3-0) = 3$$

$$\Rightarrow \frac{81}{4} = c^3(3)$$

$$\Rightarrow \frac{27}{4} = c^3$$

$$\Rightarrow c = \sqrt[3]{\frac{27}{4}} = \sqrt[3]{\frac{3}{4}} \approx \boxed{1.889}$$

(#49). $f(x) = \frac{x^2}{4} [0, 6]$

$$\int_0^6 \frac{x^2}{4} dx = \left[\frac{x^3}{12} \right]_0^6$$

$$= \frac{(6)^3}{12}$$

$$= 18$$

 $\stackrel{=0}{=}$

$$18 = \frac{c^2}{4}(6)$$

$$\Rightarrow c^2 = 12$$

$$\Rightarrow c = \pm \sqrt{12} \quad \text{choose the one in } [0, 6] \Rightarrow$$

$$f(c) = \frac{c^2}{4} \quad (b-a) = (6-0) = 6$$

$$c = \sqrt{12}$$

$$\approx 3.46$$