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Homework Solutions

(9). $\int_{-1}^{0}(2x-1)\,dx = \left[\frac{2x^2}{2}-x\right]_{-1}^{0} = \left[x^2-x\right]_{-1}^{0}$

$= ((0)^2-0)-((-1)^2-(-1))$

$= 0-(1+1) = \boxed{-2}$

(11). $\int_{-1}^{1}(t^2-5)\,dt = \left[\frac{t^3}{3}-5t\right]_{-1}^{1}$

$= \left(\frac{(1)^3}{3}-5(1)\right)-\left(\frac{(-1)^3}{3}-5(-1)\right)$

$= \left(\frac{1}{3}-5\right)-\left(\frac{-1}{3}+5\right) = \frac{1}{3}-5+\frac{1}{3}-5 = \frac{2}{3}-10 = \frac{2}{3}-\frac{30}{3}$

$= \boxed{\frac{-28}{3}}$

(13). $\int_{0}^{1}(2t-1)^2\,dt = \int_{0}^{1}(4t^2-4t+1)\,dt$

$= \left[\frac{4t^3}{3}-\frac{4t^2}{2}+t\right]_{0}^{1}$

$= \left(\frac{4(1)^3}{3}-2(1)^2+(1)\right)-(0-0+0) = \frac{4}{3}-2+1$

$= \frac{4}{3}-1 = \frac{4}{3}-\frac{3}{3} = \boxed{\frac{1}{3}}$.

(15). $\int_{1}^{2}\left(\frac{3}{x^2}-1\right)dx = \int_{1}^{2}(3x^{-2}-1)\,dx$

$= \left[\frac{3x^{-1}}{-1}-x\right]_{1}^{2}$

$= (-3(2)^{-1}-(2))-(-3(1)^{-1}-(1))$

$= \left(-\frac{3}{2}-2\right)-(-3-1) = -\frac{3}{2}-2+4 = -\frac{3}{2}+2 = -\frac{3}{2}+\frac{4}{2}$

$= \boxed{\frac{1}{2}}$.

(17). $\int_{1}^{4}\frac{u-2}{\sqrt{u}}\,du = \int_{1}^{4}\frac{u-2}{u^{1/2}}\,du$ SPLIT THE NUMERATOR

$= \int_{1}^{4}\left(\frac{u}{u^{1/2}}-\frac{2}{u^{1/2}}\right)du$

$= \int_{1}^{4}(u^{1/2}-2u^{-1/2})\,du$

$= \left[\frac{u^{3/2}}{3/2}-\frac{2u^{1/2}}{1/2}\right]_{1}^{4}$

$\Rightarrow = \left(\frac{2}{3}(4)^{3/2}-4(4)^{1/2}\right)-\left(\frac{2}{3}(1)^{3/2}-4(1)^{1/2}\right)$

$= \left(\frac{2}{3}(8)-8\right)-\left(\frac{2}{3}-4\right)$

$= \frac{16}{3}-8-\frac{2}{3}+4$

$= 14/3-4 = \frac{14}{3}-\frac{12}{3} = \boxed{2/3}$.

(19). $\int_{-1}^{1}(\sqrt[3]{t}-2)\,dt = \int_{-1}^{1}(t^{1/3}-2)\,dt$

$= \left[\frac{t^{4/3}}{4/3} - 2t\right]_{-1}^{1}$

$= \left(\frac{3}{4}(1)^{4/3} - 2(1)\right) - \left(\frac{3}{4}(-1)^{4/3} - 2(-1)\right)$

$= \left(\frac{3}{4} - 2\right) - \left(\frac{3}{4} + 2\right) = \frac{3}{4} - \frac{3}{4} - 2 - 2 = \boxed{-4}$

(21). $\int_{0}^{1} \frac{x - \sqrt{x}}{3}\,dx = \int_{0}^{1}\left(\frac{x}{3} - \frac{x^{1/2}}{3}\right)dx$

$= \left[\frac{x^2}{3 \cdot 2} - \frac{x^{3/2}}{3 \cdot 3/2}\right]_{0}^{1}$

$= \left(\frac{1}{6}(1)^2 - \frac{2}{9}(1)^{3/2}\right) - \left(\frac{1}{6}(0) - \frac{2}{9}(0)^{3/2}\right) = \frac{1}{6} - \frac{2}{9} = \frac{3}{18} - \frac{4}{18} = \boxed{\frac{-1}{18}}$

OR (ALTERNATIVE)

$= \frac{1}{3}\int_{0}^{1}(x - x^{1/2})\,dx$

$= \frac{1}{3}\left[\frac{x^2}{2} - \frac{x^{3/2}}{3/2}\right]_{0}^{1}$

$= \frac{1}{3}\left[\left(\frac{(1)^2}{2} - \frac{2(1)^{3/2}}{3}\right) - (0 - 0)\right] = \frac{1}{3}\left[\frac{1}{2} - \frac{2}{3}\right] = \frac{1}{3}\left[\frac{3}{6} - \frac{4}{6}\right] = \frac{1}{3}\left[-\frac{1}{6}\right]$

$= \boxed{\frac{-1}{18}}$

(23). $\int_{-1}^{0}(t^{1/3} - t^{2/3})\,dt = \left[\frac{t^{4/3}}{4/3} - \frac{t^{5/3}}{5/3}\right]_{-1}^{0}$

$= (0 - 0) - \left(\frac{3(-1)^{4/3}}{4} - \frac{3(-1)^{5/3}}{5}\right)$

$= (0) - \left(\frac{3}{4} - \frac{-3}{5}\right) = -\left(\frac{3}{4} + \frac{3}{5}\right) = -\left(\frac{15}{20} + \frac{12}{20}\right)$

$= \boxed{-\frac{27}{20}}$

(25). $\int_0^5 |2x-5|\,dx \Rightarrow$ the cusp is at $2x-5=0$
$\Rightarrow x = 5/2$

so we must divide into two integrals

YOU MAY <u>NOT</u> INTEGRATE ACROSS A <u>CUSP</u>!

so $\int_0^5 |2x-5|\,dx = \int_0^{5/2} |2x-5|\,dx + \int_{5/2}^5 |2x-5|\,dx$

$= \int_0^{5/2} -(2x-5)\,dx + \int_{5/2}^5 (2x-5)\,dx$ ← From the definition of abs. value

$= \int_0^{5/2} (-2x+5)\,dx + \int_{5/2}^5 (2x-5)\,dx$

$= \left[-\frac{\cancel{2}x^2}{\cancel{2}} + 5x\right]_0^{5/2} + \left[\frac{\cancel{2}x^2}{\cancel{2}} - 5x\right]_{5/2}^5$

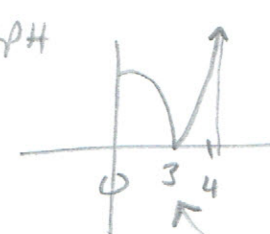
BE CAREFUL!

$= \left[\left(-(5/2)^2 + 5(5/2)\right) - (0+0)\right] + \left[\left((5)^2 - 5(5)\right) - \left(\left(\frac{5}{2}\right)^2 - 5(5/2)\right)\right]$

$= \left[\left(-\frac{25}{4} + \frac{25}{2}\right) - (0)\right] + \left[(25-25) - \left(\frac{25}{4} - \frac{25}{2}\right)\right]$

$= \left[\left(\frac{25}{4}\right) - (0)\right] + \left[(0) - \left(-\frac{25}{4}\right)\right] = \frac{25}{4} + \frac{25}{4} = \frac{50}{4} = \boxed{\frac{25}{2}}$

(#27) $\int_0^4 |x^2-9|\,dx \Rightarrow$ cusp at $x=3 \Rightarrow$ GRAPH



cusp at $x=3$

$= \int_0^3 |x^2-9|\,dx + \int_3^5 |x^2-9|\,dx$

$= \int_0^3 -(x^2-9)\,dx + \int_3^5 (x^2-9)\,dx$

$= \int_0^3 (-x^2+9)\,dx + \int_3^5 (x^2-9)\,dx$

<u>cont</u>.

$$= \left[-\frac{x^3}{3} + 9x\right]_0^3 + \left[\frac{x^3}{3} - 9x\right]_3^5$$

careful!

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$$= \left[\left(-\frac{(3)^3}{3} + 9(3)\right) - \left(-\frac{(0)^3}{3} + 9(0)\right)\right] + \left[\left(\frac{(5)^3}{3} - 9(5)\right) - \left(\frac{(3)^3}{3} - 9(3)\right)\right]$$

$$= \left[\left(-\frac{27}{3} + 27\right) - (0+0)\right] + \left[\left(\frac{125}{3} - 45\right) - \left(\frac{27}{3} - 27\right)\right]$$

$$= \left[(-9+27) - (0)\right] + \left[\left(\frac{125}{3} - 45 - 9 + 27\right)\right]$$

$$= [18] + \left[\frac{125}{3} - 27\right] = 18 + \frac{125}{3} - 27 = \frac{125}{3} - 9 = \frac{125}{3} - \frac{27}{3} = \boxed{\frac{98}{3}}$$

$64/3$

#29. $$\int_0^{\pi} (\sin(x) - 7)\,dx = \left[-\cos(x) - 7x\right]_0^{\pi}$$

$$= (-\cos(\pi) - 7(\pi)) - (-\cos(0) - 7(0))$$

$$= (-(-1) - 7\pi) - (-1 - 0) = 1 - 7\pi + 1 = \boxed{2 - 7\pi}$$

31. $$\int_0^{\pi/4} \left(\frac{1 - \sin^2(\theta)}{\cos^2(\theta)}\right) d\theta =$$

$$= \int_0^{\pi/4} \left(\frac{\cos^2(\theta)}{\cos^2(\theta)}\right) d\theta = \int_0^{\pi/4} 1\,d\theta = [\theta]_0^{\pi/4}$$

$$= (\pi/4) - (0) = \boxed{\pi/4}$$

33. $$\int_{-\pi/6}^{\pi/6} (\sec^2(x))\,dx = [\tan(x)]_{-\pi/6}^{\pi/6}$$

$$= (\tan(\pi/6)) - (\tan(-\pi/6))$$

$$= \frac{1}{\sqrt{3}} - \left(-\frac{1}{\sqrt{3}}\right)$$

$\frac{1/2}{\sqrt{3}/2}$

$$= \boxed{\frac{2}{\sqrt{3}}} \overset{\text{OR}}{=} \boxed{\frac{2\sqrt{3}}{3}}$$

#35. $\int_{-\pi/3}^{\pi/3} 4\sec(\theta)\tan(\theta)\,d\theta =$

$= \left[4\sec(\theta)\right]_{-\pi/3}^{\pi/3}$

$= \left(4\sec(\pi/3)\right) - \left(4\sec(-\pi/3)\right)$

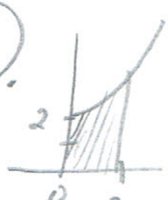
$= \left(4 \cdot \frac{1}{\cos(\pi/3)}\right) - \left(4 \cdot \frac{1}{\cos(-\pi/3)}\right)$

$= \left(4 \cdot \frac{1}{1/2}\right) - \left(4 \cdot \frac{1}{1/2}\right) = 8 - 8 = \boxed{0}$

#37. $A = \int_0^1 (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \left(\frac{(1)^2}{2} - \frac{(1)^3}{3}\right) - (0 - 0) = \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$

#39. $A = \int_0^{\pi/2} \cos(x)\,dx = \left[\sin(x)\right]_0^{\pi/2} = (\sin(\pi/2)) - (\sin(0)) = 1 - 0 = \boxed{1}$

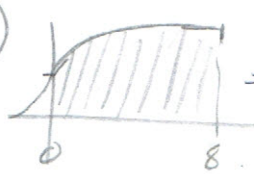
#41.



$\Rightarrow A = \int_0^2 (5x^2 + 2)\,dx = \left[\frac{5x^3}{3} + 2x\right]_0^2 = \left(\frac{5(2)^3}{3} + 2(2)\right) - (0 - 0)$

$= \frac{40}{3} + 4 = \frac{40}{3} + \frac{12}{3} = \boxed{\frac{52}{3}}$

#43.



$\Rightarrow A = \int_0^8 (1 + x^{1/3})\,dx$

$= \left[x + \frac{x^{4/3}}{4/3}\right]_0^8 = \left(8 + \frac{3}{4}(8)^{4/3}\right) - (0 + 0) = \left(8 + \frac{3}{4}(16)\right) - (0)$

$= 8 + 12 = \boxed{20}$

#45.

$\Rightarrow A = \int_0^4 (-x^2 + 4x)\,dx$

$= \left[\frac{-x^3}{3} + \frac{4x^2}{2}\right]_0^4$

$= \left(\frac{-(4)^3}{3} + \frac{4(4)^2}{2}\right) - (0 + 0)$

$= \left(-\frac{64}{3} + 32\right)$

$= \left(-\frac{64}{3} + \frac{96}{3}\right)$

$= \boxed{\frac{32}{3}}$