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Homework solutions

⑨ $\int (1+6x)^4 (6)\, dx = \int u^4\, du$

Let $u = 1+6x$
$\Rightarrow du = 6\, dx$

$= \frac{u^5}{5} + C$

$= \boxed{\frac{(1+6x)^5}{5} + C}$

DO NOT SIMPLIFY your result!!

Except for minor bits like this

⑪. $\int \sqrt{25-x^2}\,(-2x)\, dx$

Rewrite

$\int (25-x^2)^{1/2}(-2x)\, dx = \int (u)^{1/2}\, du$

Let $u = 25-x^2$
$du = -2x\, dx$

$= \frac{u^{3/2}}{3/2} + C$

$= \boxed{\frac{2}{3}(25-x^2)^{3/2} + C}$

⑬. $\int x^3 (x^4+3)^2\, dx = \frac{1}{4}\int u^2\, dx$

Let $u = x^4+3$
$\Rightarrow du = 4x^3\, dx$
$\Rightarrow \frac{1}{4} du = x^3\, dx$

$= \frac{1}{4} \cdot \frac{u^3}{3} + C$

$= \boxed{\frac{1}{4}\left(\frac{(x^4+3)^3}{3}\right) + C}$

(15). $\int x^2(2x^3-1)^4\,dx$ $= \frac{1}{6}\int u^4\,du$

Let $u = 2x^3-1$

$\Rightarrow du = 6x^2\,dx$

$\Rightarrow \frac{1}{6}\,du = x^2\,dx \leftarrow$

$= \frac{1}{6}\cdot\frac{u^5}{5} + C$

$= \frac{1}{6}\cdot\frac{(2x^3-1)^5}{5} + C$

$= \boxed{\frac{1}{30}(2x^3-1)^5 + C}$

simplifying minor bits like this is <u>optional</u>

(17) $\int t\sqrt{t^2+2}\,dt$

Rewrite

$\int t(t^2+2)^{1/2}\,dt$ $= \frac{1}{2}\int u^{1/2}\,du$

Let $u = t^2+2$

$\Rightarrow du\ 2t\,dt$

$\Rightarrow \frac{1}{2}\,du = t\,dt \leftarrow$

$= \frac{1}{2}\,\frac{u^{3/2}}{3/2} + C$

$= \frac{1}{2}\cdot\frac{2}{3}(t^2+2)^{3/2} + C = \boxed{\frac{1}{3}(t^2+2)^{3/2} + C}$

(19). $\int 5x\sqrt{1-x^2}\,dx$

Rewrite

$\int 5x(1-x^2)^{1/2}\,dx$ $= -\frac{5}{2}\int u^{1/2}\,du$

Let $u = 1-x^2$

$\Rightarrow du = -2x\,dx$

$\Rightarrow -\frac{1}{2}\,du = x\,dx$

$\Rightarrow -\frac{5}{2}\,du = 5x\,dx \leftarrow$

$= -\frac{5}{2}\,\frac{u^{3/2}}{3/2} + C$

$= -\frac{5}{2}\cdot\frac{2}{3}(1-x^2)^{3/2} + C = \boxed{-\frac{5}{3}(1-x^2)^{3/2} + C}$

(21). $\int \frac{7x}{(1-x^2)^3}\,dx \Rightarrow$ THERE IS <u>NO</u> QU<u>OTIENT</u> <u>RULE</u> FOR INTEGRALS!!

REWRITE:

$= \int 7x(1-x^2)^{-3}\,dx$

Let $u = 1-x^2\,dx$

$\Rightarrow du = -2x\,dx$

$\Rightarrow -\frac{1}{2}\,du = x\,dx$

$\Rightarrow -\frac{7}{2}\,du = 7x\,dx \leftarrow$

$= -\frac{7}{2}\int u^{-3}\,du$

$= -\frac{7}{2}\,\frac{u^{-2}}{-2} + C$

$= \boxed{\frac{7}{4}(1-x^2)^{-2} + C}$

(23). $\int \frac{x^2}{(1+x^3)^2}\,dx$

Rewrite

$\int x^2(1+x^3)^{-2} = \frac{1}{3}\int u^{-2}\,du$

Let $u = 1+x^3$

$\Rightarrow du = 3x^2\,dx$

$\Rightarrow \frac{1}{3}du = x^2\,dx$

$= \frac{1}{3}\frac{u^{-1}}{-1} + C$

$= \boxed{-\frac{1}{3}(1+x^3)^{-1} + C}$

(25). $\int \frac{x}{\sqrt{1-x^2}}\,dx$ REWRITE $= \int x(1-x^2)^{-1/2}\,dx = -\frac{1}{2}\int u^{-1/2}\,dx$

Let $u = 1-x^2$

$\Rightarrow du = -2x\,dx$

$\Rightarrow -\frac{1}{2}du = x\,dx$

$= -\frac{1}{2}\frac{u^{1/2}}{1/2} + C$

$= \boxed{-\frac{1}{2}\cdot\frac{2}{1}(1-x^2)^{1/2} + C}$

(27). $\int \left(1+\frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)dt$

Rewrite:

$= \int (1+t^{-1})^3(t^{-2})\,dt = -\int u^3\,du$

Let $u = 1+t^{-1}$

$\Rightarrow du = -t^{-2}\,dt$

$\Rightarrow -du = t^{-2}\,dt$

$= -\frac{u^4}{4} + C$

$= \boxed{-\frac{(1+t^{-1})^4}{4} + C}$

(29). $\int \frac{1}{\sqrt{2x}}\,dx$

Rewrite

$= \int (2x)^{-1/2}\,dx = \frac{1}{2}\int u^{-1/2}\,du$

Let $u = 2x$

$\Rightarrow du = 2\,dx$

$\Rightarrow \frac{1}{2}du = dx$

$= \frac{1}{2}\frac{u^{1/2}}{\frac{1}{2}} + C$

$= \frac{1}{2}\cdot\frac{2}{1}(2x)^{1/2} + C = \boxed{(2x)^{1/2} + C}$

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(39). $\int \pi \sin(\pi x)\,dx = \int \sin(u)\,du$

Let $u = \pi x$
$du = \pi\,dx$

$= -\cos(u) + C$

$= \boxed{-\cos(\pi x) + C}$

(41). $\int \cos(6x)\,dx = \frac{1}{6}\int \cos(u)\,du$

Let $u = 6x$
$\Rightarrow du\ 6\,dx$
$\Rightarrow \frac{1}{6}du = dx$

$= \frac{1}{6}\sin(u) + C$

$= \boxed{\frac{1}{6}\sin(6x) + C}$

(43) $\int \frac{1}{\theta^2}\cos\left(\frac{1}{\theta}\right)d\theta$

Rewrite

$\int \theta^{-2}\cos(\theta^{-1})\,d\theta = -\int \cos(u)\,du$

Let $u = \theta^{-1}$
$\Rightarrow du = -\theta^{-2}d\theta$
$\Rightarrow -du = \theta^{-2}d\theta$

$= -\sin(u) + C$

$= \boxed{-\sin(\theta^{-1}) + C}$

(45). $\int \sin(2x)\cos(2x)\,dx$ ← THERE ARE TWO OPTIONS
I WILL WORK BOTH.

(I). Let $u = \sin(2x)$
$\Rightarrow du = \cos(2x)(2)\,dx$
$\Rightarrow \frac{1}{2}du = \cos(2x)\,dx$

$= \frac{1}{2}\int u\,du$

$= \frac{1}{2}\frac{u^2}{2} + C$

$= \boxed{\frac{1}{4}\sin^2(2x) + C}$

(II) Let $u = \cos(2x)$
$\Rightarrow du = -\sin(2x)(2)\,dx$
$\Rightarrow -\frac{1}{2}du = \sin(2x)\,dx$

$= -\frac{1}{2}\int u\,du$

$= -\frac{1}{2}\cdot\frac{u^2}{2} + C_1$

$= \boxed{-\frac{1}{4}\cos^2(2x) + C_1}$

Hint: The key is the "C" constant

EITHER OF THESE IS OK. AN INTERESTING EXERCISE IS TO SHOW THESE TWO ARE EQUAL.

(53). $\int x\sqrt{x+6}\,dx$ (double linear scenario)

Rewrite:

$\int x(x+6)^{1/2}dx$

Let $u=x+6 \Rightarrow u-6=x$

$du=dx$

$= \int (u-6)u^{1/2}\,du$ ← NOW DISTRIBUTE

$= \int (u^{3/2}-6u^{1/2})\,du$

$= \dfrac{u^{5/2}}{5/2} - \dfrac{6u^{3/2}}{3/2} + C$

$= \boxed{\frac{2}{5}(x+6)^{5/2} - 6\left(\frac{2}{3}\right)(x+6)^{3/2} + C}$

(54) $\int x\sqrt{3x-4}\,dx$ Double linear

Rewrite

$\int x(3x-4)^{1/2}dx$

Let $u=3x-4 \Rightarrow u-4=3x$

$\Rightarrow du=3dx \qquad \Rightarrow \frac{1}{3}(u-4)=x$

$\Rightarrow \frac{1}{3}du=dx$

$= \frac{1}{3}\int \frac{1}{3}(u-4)u^{1/2}\,du$

$= \frac{1}{9}\int (u^{3/2}-4u^{1/2})\,du$

$= \frac{1}{9}\left[\dfrac{u^{5/2}}{5/2} - \dfrac{4u^{3/2}}{3/2}\right] + C$

$= \boxed{\frac{1}{9}\left[\frac{2}{5}(3x-4)^{5/2} - (4)\left(\frac{2}{3}\right)(3x-4)^{3/2}\right] + C}$

(61). $\int_{-1}^{1} x(x^2+1)^3\,dx$

Let $u=x^2+1$

$du=2x\,dx$

$\frac{1}{2}du=x\,dx$

$= \frac{1}{2}\int_2^2 u^3\,du = \boxed{0}$ (since upper and lower limits of integration are equal.)

Limits

upper $u=(1)^2+1=2$

lower $u=(-1)^2+1=2$

(63) $\int_1^2 2x^2\sqrt{x^3+1}\,dx$

Rewrite:

$\int_1^2 2x^2(x^3+1)^{1/2}\,dx$

Let $u = x^3+1$

$\Rightarrow du = 3x^2\,dx$

$\Rightarrow \frac{1}{3}du = x^2dx$

$\Rightarrow \frac{2}{3}du = 2x^2dx$

Limits

upper

$u = (2)^3+1 = 9$

Lower

$u = (1)^3+1 = 2$

$= \frac{2}{3}\int_2^9 u^{1/2}\,du$

$= \frac{2}{3}\left[\frac{u^{3/2}}{3/2}\right]_2^9$

$= \frac{2}{3}\left[\frac{2}{3}u^{3/2}\right]_2^9$

$= \frac{4}{9}\left[(9)^{3/2}-(2)^{3/2}\right]$

$= \boxed{\frac{4}{9}\left[27-\sqrt{8}\right]} \approx \boxed{10.74}$

(65) $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$

$= \int_0^4 (2x+1)^{-1/2}\,dx$

Let $u = 2x+1$

$du = 2dx$

$\frac{1}{2}du = dx$

Limits

upper

$u = 2(4)+1$

$= 9$

Lower

$u = 2(0)+1$

$= 1$

$= \frac{1}{2}\int_1^9 u^{-1/2}\,du$

$= \frac{1}{2}\left[\frac{u^{1/2}}{1/2}\right]_1^9$

$= \cancel{\frac{1}{2}}\left[\cancel{\frac{2}{1}}\,u^{1/2}\right]_1^9$

$= (9)^{1/2}-(1)^1$

$= 3-1$

$= \boxed{2}$

(67) $\int_1^9 \frac{1}{\sqrt{x}\,(1+\sqrt{x})^2}\,dx$

$= \int_1^9 x^{-1/2}(1+x^{1/2})^{-2}\,dx \Longrightarrow = 2\int_2^4 u^{-2}\,du = 2\left[\frac{u^{-1}}{-1}\right]_2^4$

Let $u = 1+x^{1/2}$

$\Rightarrow du = \frac{1}{2}x^{-1/2}dx$

$\Rightarrow 2du = x^{-1/2}dx$

Limits

upper $u = 1+(9)^{1/2} = 4$ Lower $u = 1+(1)^{1/2} = 2$

$= -2\left[\frac{1}{u}\right]_2^4$

$= -2\left[\left(\frac{1}{4}\right)-\left(\frac{1}{2}\right)\right]$

$= -2\left[-\frac{1}{4}\right]$

$= \boxed{\frac{1}{2}}$