

43) $f(x) = \ln(3x) \Rightarrow f'(x) = \boxed{\frac{1}{(3x)} (3)}$

DO NOT
SIMPLIFY ON
THE EXAM!!

45) $f(x) = \ln(x^2+3)$
 $\Rightarrow f'(x) = \boxed{\frac{1}{(x^2+3)} (2x)}$ ← EITHER IS OK
OR $\boxed{\frac{2x}{x^2+3}}$

47) $y = (\ln(x))^4 \Rightarrow f'(x) = \boxed{4(\ln(x))^3 \left(\frac{1}{x}\right)}$

49) $y = \ln(t+1)^2 \Rightarrow y' = \boxed{\frac{1}{(t+1)^2} (2(t+1)(1))}$

OR
Rewrite: $y = 2\ln(t+1) \Rightarrow y' = \boxed{2\left(\frac{1}{t+1}\right)(1)}$

5) $y = \ln(x\sqrt{x^2+1})$

rewrite

$y = \ln(x \cdot (x^2+1)^{1/2}) \Rightarrow y' = \boxed{\frac{1}{x(x^2+1)^{1/2}} \left(x \left(\frac{1}{2}(x^2+1)^{-1/2} (2x) \right) + (x^2+1)^{1/2} (1) \right)}$

OR
Rewrite:

$y = \ln(x) + \ln(x^2+1)^{1/2}$

$= \ln(x) + \frac{1}{2}\ln(x^2+1)$

use properties
of Logs

$\Rightarrow y' = \boxed{\frac{1}{x} + \frac{1}{2} \cdot \frac{1}{x^2+1} (2x)}$

53) $f(x) = \ln\left(\frac{x}{x^2+1}\right)$

→ Rewrite:

$$f(x) = \ln(x) - \ln(x^2+1)$$

$$\Rightarrow f'(x) = \frac{1}{x} - \frac{1}{(x^2+1)}(2x)$$

$$= \boxed{\frac{1}{x} - \frac{2x}{x^2+1}}$$

55) $g(t) = \frac{\ln(t)}{t^2}$

Rewrite

$$g(t) = t^{-2} \cdot \ln(t) \Rightarrow g'(t) = \boxed{t^{-2} \left(\frac{1}{t}\right) + \ln(t)(-2t^{-3})}$$

57) $y = \ln(\ln(x^2))$

$$\Rightarrow y' = \boxed{\frac{1}{\ln(x^2)} \cdot \frac{1}{(x^2)} (2x)}$$

Rewrite

or $y = \ln(2\ln(x))$

$$\Rightarrow y' = \boxed{\left(\frac{1}{2\ln(x)}\right) \left(\frac{2}{x}\right)}$$

59) $y = \ln\left(\sqrt{\frac{x+1}{x-1}}\right)$ this is Much Easier if you re-write

Rewrite

$$y = \ln\left(\frac{x+1}{x-1}\right)^{1/2} = \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right) = \frac{1}{2} [\ln(x+1) - \ln(x-1)]$$

$$\text{so } y' = \boxed{\frac{1}{2} \left[\frac{1}{(x+1)} - \frac{1}{(x-1)} \right]}$$

61) $f(x) = \ln\left(\frac{\sqrt{4+x^2}}{x^2}\right)$ AGAIN Rewrite using properties of logs

$$\Rightarrow f(x) = \ln((4+x^2)^{1/2}) - \ln(x^2)$$

$$= \frac{1}{2} \ln(4+x^2) - 2\ln(x)$$

$$\text{so } f'(x) = \boxed{\frac{1}{2} \left(\frac{1}{4+x^2} \right) (2x) - \frac{2}{x}}$$

(67) $y = \ln(x^4)$ $(1, 0) \Rightarrow y - 0 = \underline{\hspace{2cm}} (x - 1)$
 Re-write need slope

$$y = 4 \ln(x) \Rightarrow y' = 4 \cdot \frac{1}{x}$$

$$\text{so } [y']_{x=1} = \left[\frac{4}{x} \right]_{x=1} = \frac{4}{1} = 4$$

$$\Rightarrow y - 0 = 4(x - 1)$$

$$\Rightarrow \boxed{y = 4x - 4}$$

(69) $f(x) = 3x^2 - \ln(x)$ $(1, 3) \Rightarrow y - 3 = \underline{\hspace{2cm}} (x - 1)$

$$\Rightarrow f'(x) = 6x - \frac{1}{x} \Rightarrow f'(1) = 6(1) - \frac{1}{1}$$

$$= 6 - 1 = 5$$

$$\xrightarrow{\text{so}} y - 3 = 5(x - 1)$$

$$\Rightarrow y - 3 = 5x - 5$$

$$\Rightarrow \boxed{y = 5x - 2}$$

(73) $y = x^3 \cdot \ln(x^4)$ $(-1, 0) \Rightarrow y - 0 = \underline{\hspace{2cm}} (x - (-1))$

$$\Rightarrow y' = (x^3) \left(\frac{1}{x^4} (4x^3) \right) + (\ln(x^4)) (3x^2)$$

$$\Rightarrow [y']_{x=-1} = ((-1)^3) \left(\frac{1}{(-1)^4} (4(-1)^3) \right) + (\ln((-1)^4)) (3(-1)^2)$$

$$= (-1)(1)(-4) + (0)(3)$$

$$= 4$$

$$\xrightarrow{\text{so}} y - 0 = 4(x + 1)$$

$$\Rightarrow \boxed{y = 4x + 4}$$