Calc I sec 5.2 pg 330
Homework Solutions

(5) $\int \frac{5}{x}\,dx = 5\int \frac{1}{x}\,dx = 5\ln|x| + C$

(7) $\int \frac{1}{2x+5}\,dx$ Let $u = 2x+5$ $\Rightarrow du = 2dx$ $\Rightarrow \frac{1}{2}du = dx$ $= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|2x+5| + C}$

(9) $\int \frac{x}{x^2-3}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$

Let $u = x^2-3$
$\Rightarrow du = 2x\,dx$
$\Rightarrow \frac{1}{2}du = x\,dx$

$= \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|x^2-3| + C}$

(11) $\int \frac{4x^3+3}{x^4+3x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|x^4+3x| + C}$

Let $u = x^4+3x$
$du = (4x^3+3)\,dx$

(13) $\int \frac{x^2-7}{7x}\,dx$ SPLIT THE NUMERATOR $= \int \left(\frac{x^2}{7x} - \frac{7}{7x}\right)dx$

$= \int \left(\frac{1}{7}x - \frac{1}{x}\right)dx = \boxed{\frac{1}{7}\frac{x^2}{2} - \ln|x| + C}$ ← OPTIONAL simplifying

$= \boxed{\frac{x^2}{14} - \ln|x| + C}$

(15) $\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx = \frac{1}{3}\int \frac{1}{u}\,du$

Let $u = x^3+3x^2+9x$
$\Rightarrow du = (3x^2+6x+9)\,dx$
$= 3(x^2+2x+3)\,dx$
$\Rightarrow \frac{1}{3}du = (x^2+2x+3)\,dx$

$= \frac{1}{3}\ln|u| + C$
$= \boxed{\frac{1}{3}\ln|x^3+3x^2+9x| + C}$

(17) $\int \frac{x^2-3x+2}{x+1}\,dx$ If degree of numerator $\geq$ degree of denominator pg 2 two →

simplify the integrand

→ by long division:

$4-6+2$

$$\begin{array}{r} x-4+\frac{6}{x+1} \\ x+1\overline{)\,x^2-3x+2} \\ \ominus\, x^2+x \\ \hline -4x+2 \\ \oplus\, -4x-4 \\ \hline +6 \end{array}$$

so

$$= \int\left(x-4+\frac{6}{x+1}\right)dx = \int(x-4)\,dx + 6\int\frac{1}{x+1}\,dx$$

Let $u = x+1$

$du = dx$

$$= \frac{x^2}{2} - 4x + C_1 + 6\int \frac{1}{u}\,du$$

$$= \frac{x^2}{2} - 4x + C_1 + 6\ln|u| + C_2$$

$$= \boxed{\frac{x^2}{2} - 4x + 6\ln|x+1| + C}$$

(18)

$$\int \frac{x^3-3x^2+5}{x-3}\,dx = \int\left(x^2+\frac{5}{x-3}\right)dx$$

$$\begin{array}{r} x^2+\frac{5}{x-3} \\ x-3\overline{)\,x^3-3x^2+0x+5} \\ \ominus\, x^3 \oplus 3x^2 \\ \hline +0x+5 \end{array}$$

$$= \int x^2\,dx + 5\int\frac{1}{x-3}\,dx$$

Let $u = x-3$

$du = dx$

$$= \int x^2\,dx + 5\int\frac{1}{u}\,du$$

$$= \frac{x^3}{3} + 5\ln|u| + C$$

$$= \boxed{\frac{x^3}{3} + 5\ln|x-3| + C}$$

(21) SKIP THIS ONE.

(23) $\displaystyle\int \frac{(\ln(x))^2}{x}\,dx = \int u^2\,du = \frac{u^3}{3}+C = \boxed{\frac{(\ln(x))^3}{3}+C}$

Let $u=\ln(x)$

$du = \frac{1}{x}dx$

(25) $\displaystyle\int \frac{1}{\sqrt{x}(1-3\sqrt{x})}\,dx \underset{\text{Rewrite}}{=} \int \frac{x^{-1/2}}{(1-3x^{1/2})}\,dx = -\frac{2}{3}\int \frac{1}{u}\,du$

Let $u=1-3x^{1/2}$

$\Rightarrow du = -\frac{3}{2}x^{-1/2}dx$

$\Rightarrow -\frac{2}{3}du = x^{-1/2}dx$

$= -\frac{2}{3}\ln|u|+C$

$= \boxed{-\frac{2}{3}\ln|1-3x^{1/2}|+C}$

(27) $\displaystyle\int \frac{6x}{(x-5)^2}\,dx = 6\int \frac{x}{(x-5)}\,dx$ "Double linear"

Let $u=x-5 \Rightarrow u+5=x$

$du=dx$

so we have:

$\displaystyle = 6\int \frac{u+5}{u}\,du$ Now split the numerator

$\displaystyle = 6\int\left(\frac{u}{u}+\frac{5}{u}\right)du$

$\displaystyle = 6\int\left(1+\frac{5}{u}\right)du$

$= 6(u+5\ln|u|)+C$

$= \boxed{6(x-5+5\ln|x-5|)+C}$

pg 4ours

(51) $\int_0^4 \frac{5}{3x+1}\,dx$

Let $u = 3x+1$ Limits

$\Rightarrow du = 3dx$ upper $u = 3(4)+1 = 13$

$\Rightarrow \frac{1}{3}du = dx$ Lower $u = 3(0)+1 = 1$

$\Rightarrow \frac{5}{3}du = 5dx$

$$= \frac{5}{3}\int_1^{13} \frac{1}{u}\,du = \left[\frac{5}{3}\ln|u|\right]_1^{13} = \frac{5}{3}\left[\ln(13) - \overset{0}{\overset{\shortparallel}{\ln(1)}}\right] = \boxed{\frac{5}{3}\ln(13)}$$

(53) $\int_1^e \frac{(1+\ln(x))^2}{x}\,dx$

Let $u = 1+\ln(x)$ Limits

$du = \frac{1}{x}dx$ upper $u = 1+\ln(e) = 1+1 = 2$

lower $u = 1+\ln(1) = 1+0 = 1$

$$= \int_1^2 u^2\,du = \left[\frac{u^3}{3}\right]_1^2 = \left(\frac{(2)^3}{3}\right) - \left(\frac{(1)^3}{3}\right) = \frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}}$$

(55) $\int_0^2 \frac{x^2-2}{x+1}\,dx = \int_0^2 \left(x-1-\frac{1}{x+1}\right)dx$

DIVIDE

$$\begin{array}{r} x-1-\frac{1}{x+1} \\ x+1\overline{\big)\,x^2+0x-2} \\ -x^2-x \\ \hline -x-2 \\ +x+1 \\ \hline -1 \end{array}$$

$= \int_0^2 (x-1)\,dx - \int_0^2 \frac{1}{x+1}\,dx$ Let $u = x+1$, $du = dx$ Limits upper $u = 2+1 = 3$, lower $u = 0+1 = 1$

$$= \left[\frac{x^2}{2} - x\right]_0^2 - \int_1^3 \frac{1}{u}\,du$$

$$= \left[\frac{x^2}{2} - x\right]_0^2 - \left[\ln|u|\right]_1^3$$

$$= \left[\left(\frac{(2)^2}{2} - (2)\right) - \left(\frac{(0)^2}{2} - 0\right)\right] - \left[\ln|3| - \ln|1|\right]$$

$$= [2-2] - [\ln(3) - 0] = \boxed{-\ln(3)}$$

(57) $\int_1^2 \frac{1-\cos(\theta)}{\theta-\sin(\theta)}\,d\theta$ pg 5ive

Let $u=\theta-\sin(\theta)$

$du=(1-\cos(\theta))\,d\theta$

Limits

upper $u=(2)-\sin(2)=2-\sin(2)$

Lower $u=(1)-\sin(1)=1-\sin(1)$

$$=\int_{1-\sin(1)}^{2-\sin(2)} \frac{1}{u}\,du=\Big[\ln|u|\Big]_{1-\sin(1)}^{2-\sin(2)}$$

$$=\boxed{\ln|2-\sin(2)|-\ln|1-\sin(1)|}$$