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Homework solutions:

(#33) $y = e^{5x} \Rightarrow y' = \boxed{e^{5x}(5)}$ or $\boxed{5e^{5x}}$

This sort of simplifying is fine

(35) $y = e^{\sqrt{x}} \Rightarrow y = e^{x^{1/2}} \Rightarrow y' = \boxed{e^{x^{1/2}}\left(\frac{1}{2}x^{-1/2}\right)}$

But DO NOT simplify this sort of result on the exam!

(37) $y = e^{(x-4)} \Rightarrow \frac{dy}{dx} = \boxed{e^{(x-4)}(1)}$

(39) $y = e^x \ln(x)$ product rule

$\Rightarrow y' = \boxed{(e^x)\left(\frac{1}{x}\right) + (e^x)(\ln(x))}$

(41) $y = (x+1)^2 e^x \Rightarrow y' = \boxed{(x+1)^2(e^x) + (e^x)(2)(x+1)^1(1)}$

chain rule 2

(43) $g(t) = (e^{-t} + e^t)^3 \Rightarrow g'(t) = \boxed{(3)(e^{-t}+e^t)^2\left(e^{-t}(-1) + e^{(t)}(1)\right)}$

(45) $y = \ln(2 - e^{5x}) \Rightarrow \frac{dy}{dx} = \boxed{\frac{1}{(2-e^{(5x)})}\left(-e^{5x}(5)\right)}$

(47) $y = \frac{2}{e^x + e^{-x}} = 2(e^x + e^{-x})^{-1}$

$\Rightarrow y' = \boxed{(2)(-1)(e^x + e^{-x})^{-2}(e^x + e^{-x}(-1))}$

DO NOT SIMPLIFY!!

(49) $y = \frac{e^x + 1}{e^x - 1} \Rightarrow y' = \boxed{\frac{(e^x - 1)(e^x) - (e^x + 1)(e^x)}{(e^x - 1)^2}}$

Quotient rule

(#51). $y = e^{x}(\sin(x)+\cos(x))$

$\Rightarrow y' = \boxed{(e^{x})(\cos(x)-\sin(x)) + (\sin(x)+\cos(x))e^{x}}$

(91). $\int e^{5x}(5)\,dx = \int e^{u}\,du = e^{u}+C$

$= \boxed{e^{(5x)}+C}$

Let $u = 5x$

$\Rightarrow du = 5dx$

(93). $\int e^{(5x-3)}\,dx = \frac{1}{5}\int e^{u}\,du$

Let $u = 5x-3$ $\quad = \frac{1}{5}e^{u}+C = \boxed{\frac{1}{5}e^{(5x-3)}+C}$

$\Rightarrow du = 5dx$

$\Rightarrow \frac{1}{5}du = dx$

(95). $\int (2x+1)e^{(x^2+x)}\,dx = \int e^{u}\,du$

Let $u = x^2+x$ $\quad = e^{u}+C = \boxed{e^{(x^2+x)}+C}$

$\Rightarrow du = (2x+1)dx$

(97) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int e^{(x^{1/2})}(x^{-1/2})\,dx = 2\int e^{u}\,du$

Let $u = x^{1/2}$ $\quad = 2e^{u}+C = \boxed{2e^{x^{1/2}}+C}$

$\Rightarrow du = \frac{1}{2}x^{-1/2}dx$

$\Rightarrow 2du = x^{-1/2}dx$

(99) $\int \frac{e^{-x}}{1+e^{-x}}\,dx$ Let $u = 1+e^{-x}$ $\quad = -\int \frac{1}{u}\,du = -\ln|u|+C$

$\Rightarrow du = e^{-x}(-1)\,dx \Rightarrow -du = e^{-x}dx$ $\quad = \boxed{-\ln|1+e^{-x}|+C}$

(101). $\int e^x\sqrt{1-e^x}\,dx$

$= \int e^x(1-e^x)^{1/2}\,dx = -\int u^{1/2}\,du$

Let $u = 1-e^x$

$\Rightarrow du = -e^x\,dx$

$\Rightarrow -du = e^x\,dx$

$= -\dfrac{u^{3/2}}{3/2} + C = \boxed{-\frac{2}{3}(1-e^x)^{3/2} + C}$

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(103). $\int \dfrac{e^x+e^{-x}}{e^x-e^{-x}}\,dx = \int \dfrac{1}{u}\,du$

$= \ln|u| + C = \boxed{\ln|e^x - e^{-x}| + C}$

Let $u = e^x - e^{-x}$

$du = (e^x - e^{-x}(-1))\,dx$

$= (e^x + e^{-x})\,dx$

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(105). $\int \dfrac{5-e^x}{e^{2x}}\,dx \overset{\text{split the numerator}}{=} \int\left(\dfrac{5}{e^{2x}} - \dfrac{e^x}{e^{2x}}\right)dx$

$= \int 5e^{-2x}\,dx - \int e^{-x}\,dx$

Let $u = -2x$

$\Rightarrow du = -2\,dx$

$\Rightarrow \frac{5}{-2}\,du = 5\,dx$

Let $v = -x$

$\Rightarrow dv = -dx$

$\Rightarrow -dv = dx$

$= -\frac{5}{2}\int e^u\,du - \left(-\int e^v\,dv\right)$

$= -\frac{5}{2}\int e^u\,du + \int e^v\,dv = -\frac{5}{2}e^u + e^v + C$

$= \boxed{-\frac{5}{2}e^{-2x} + e^{-x} + C}$

(#107) $\int e^{-x}\tan(e^{-x})\,dx = -\int \tan(u)\,du$

Let $u = e^{-x}$

$\Rightarrow du = e^{-x}(-1)\,dx$

$\Rightarrow -du = e^{-x}\,dx$

$= -\ln|\sec(u)| + C$

$= \boxed{-\ln|\sec(e^{-x})| + C}$

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(109) $\int_0^1 e^{-2x}\,dx = \int_0^{-2} e^u\,du$

Let $u = -2x$

$\Rightarrow du = -2dx$

$\Rightarrow -\frac{1}{2}du = dx$

Limits

upper $u = -2(1) = -2$

Lower $u = -2(0) = 0$

$= [e^u]_0^{-2} = \boxed{e^{-2} - e^0}$

$= \boxed{e^{-2} - 1}$

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(111) $\int_0^1 xe^{-x^2}\,dx = -\frac{1}{2}\int_0^{-1} e^u\,du$

Let $u = -x^2$

$\Rightarrow du = -2x\,dx$

$\Rightarrow -\frac{1}{2}du = x\,dx$

Limits

upper $u = -(1)^2 = -1$

Lower $u = -(0)^2 = 0$

$= \left[-\frac{1}{2}e^u\right]_0^{-1} = \left(-\frac{1}{2}e^{-1}\right) - \left(-\frac{1}{2}e^0\right)$

$= -\frac{1}{2}e^{-1} + \frac{1}{2}(1)$

$= \boxed{-\frac{1}{2}e^{-1} + \frac{1}{2}}$

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(113) $\int_1^3 \frac{e^{3/x}}{x^2}\,dx = \int_1^3 e^{3x^{-1}} x^{-2}\,dx$

Let $u = 3x^{-1}$

$\Rightarrow du = -3x^{-2}\,dx$

$\Rightarrow -\frac{1}{3}du = x^{-2}dx$

Limits upper $u = 3(3)^{-1} = 1$

lower $u = 3(1)^{-1} = 3$

$= -\frac{1}{3}\int_3^1 e^u\,du$

$= -\frac{1}{3}[e^u]_3^1$

$= \boxed{-\frac{1}{3}[(e^1) - (e^3)]}$