TRIG EQUATION EXAMPLES
SOLUTIONS

① $2\sin(x)\cos(x)+\sin(x)=0$

FACTOR: $\sin(x)(2\cos(x)+1)=0$

$\Rightarrow \sin(x)=0$ OR $2\cos(x)+1=0$

$\Rightarrow x=0;\ x=\pi$

$\Rightarrow \cos(x)=-1/2$

Quadrant Check: QII QIII

$x_R=\pi/3$

$\Rightarrow x=2\pi/3;\ x=4\pi/3$

Sol: $\{0, \pi, 2\pi/3, 4\pi/3\}$

② $4\cos^2(x)-2=0$

$\Rightarrow \cos^2(x)=2/4=1/2$

$\Rightarrow \cos(x)=\pm\frac{1}{\sqrt{2}}$

Quadrant Check: ALL FOUR

$x_R=\pi/4 \Rightarrow \pi/4;\ 3\pi/4;\ 5\pi/4;\ 7\pi/4$

Sol: $\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$

Not allowed $x=\pi/2,\ x=3\pi/2$

③ $\sin(x)\tan^2(x)-3\sin(x)=0$

$\Rightarrow$ Factor: $\sin(x)(\tan^2(x)-3)=0$

$\Rightarrow \sin(x)=0$ OR $\tan^2(x)-3=0$

$\Rightarrow x=0,\ x=\pi$

$\Rightarrow \tan^2(x)=3$

$\Rightarrow \tan(x)=\pm\sqrt{3}$

Quadrant check: ALL FOUR

$x_R=\pi/3$

$\Rightarrow \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

Sol: $\{0, \pi, \pi/3, 2\pi/3, 4\pi/3, 5\pi/3\}$

④ $\sec(x)-2=0$

Solve for $\sec(x)$

$\Rightarrow \sec(x)=2$

change to $\cos(x)$

$\Rightarrow \cos(x)=1/2$

$\Rightarrow \cos(x)=1/2$

Quad. Chk: QI QIV

$x_R=\pi/3 \Rightarrow \pi/3, 5\pi/3$

Sol: $\{\pi/3, 5\pi/3\}$

(5). $2\sin^2(x) + 3\sin(x) + 1 = 0$

Factor: $(2\sin(x) + 1)(\sin(x) + 1) = 0$

$\Rightarrow 2\sin(x) + 1 = 0$ $\Rightarrow \sin(x) + 1 = 0$

$\Rightarrow \sin(x) = -1/2$ $\Rightarrow \sin(x) = -1$

Quad Chk QIII QIIII

$x_R = \pi/6$

so $x = 7\pi/6$ $x = 11\pi/6$ $x = 3\pi/2$

So $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}\right\}$

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(6). $10\sin^2(x) - 5 = 0$

$\Rightarrow \sin^2(x) = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \sin(x) = \pm\sqrt{1/2}$

$\Rightarrow \sin(x) = \pm\frac{1}{\sqrt{2}}$

Quad Chk: all 4

$x_R = \pi/4 \Rightarrow \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

So $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

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(7). $3\csc^2(x) - 4 = 0$

solve for $\csc^2(x)$:

$\csc^2(x) = \frac{4}{3}$

change to $\sin^2(x)$

$\Rightarrow \sin^2(x) = \frac{3}{4}$

$\Rightarrow \sin(x) = \pm\sqrt{\frac{3}{4}}$

$\Rightarrow \sin(x) = \pm\frac{\sqrt{3}}{2}$

Quad Chk: ALL FOUR

$x_R = \pi/3$ ↳ $\pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

NOT ALLOWED: $x = 0, x = \pi$

So $\left\{\pi/3, 2\pi/3, 4\pi/3, 5\pi/3\right\}$

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(8) $\sec(x)\tan(x) + \sec(x) = 0$ $\Rightarrow$ NOT ALLOWED $x = \pi/2$ $x = 3\pi/2$

FACTOR:

$\sec(x)(\tan(x) + 1) = 0$

$\Rightarrow \sec(x) = 0$ or $\tan(x) + 1 = 0$

$\Rightarrow$ NO SOL'N

$\emptyset$

$\Rightarrow \tan(x) = -1$

Quad Chk QII QIIII

$x_R = \pi/4 \Rightarrow x = \frac{3\pi}{4}$ $x = \frac{7\pi}{4}$

So $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

(9) $(\tan^2(x)-1)(\cos^2(x)-1)=0$ ⇒ Not allowed $x=\frac{\pi}{2}$, $x=\frac{3\pi}{2}$

$\Rightarrow \tan^2(x)-1=0$ OR $\cos^2(x)-1=0$

$\Rightarrow \tan^2(x)=1$ $\Rightarrow \cos^2(x)=1$

$\Rightarrow \tan(x)=\pm 1$ $\Rightarrow \cos(x)=\pm 1$

Quad Chk: All Four

$x_R=\pi/4 \Rightarrow \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$x=0$

$x=\pi$

So $\{0, \pi, \pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$

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(10) $2\cos^2(x)-\cos(x)-1=0$

FACTOR:

$(2\cos(x)+1)(\cos(x)-1)=0$

$\Rightarrow 2\cos(x)+1=0$ OR $\cos(x)-1=0$

$\Rightarrow \cos(x)=-1/2$ $\Rightarrow \cos(x)=1$

Quad chk: QII QIII

$x_R=\pi/3 \Rightarrow \frac{2\pi}{3}, \frac{4\pi}{3}$

$\Rightarrow x=0$

So $\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$

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(11) $\sin(x)\sec(x)+\frac{1}{2}\sin(x)=0$ ⇒ Not allowed $x=\frac{\pi}{2}$ $x=\frac{3\pi}{2}$

Factor: $\sin(x)\left(\sec(x)+\frac{1}{2}\right)=0$

$\Rightarrow \sin(x)=0$ OR $\sec(x)+\frac{1}{2}=0$

$x=0$ $x=\pi$

$\Rightarrow \sec(x)=-\frac{1}{2}$

$\Rightarrow \cos(x)=-2$

$\Rightarrow$ NO SOLUTION

(-2 is not in the range of $\cos(x)$)

So $\{0, \pi\}$

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(12) $\sin^2(x)+\cos(x)\sin(x)=0$

Factor

$\sin(x)(\sin(x)+\cos(x))=0$

cont

(12) Cont ⇓

$\Rightarrow \sin(x) = 0$

$\Rightarrow$ 

$\Rightarrow x = 0 \quad x = \pi$

or $\sin(x) + \cos(x) = 0$

(*) This equation (or $\sin(x) - \cos(x) = 0$) frequently arises. There are various techniques to solve it. This one is standard:

$\Rightarrow \sin(x) + \cos(x) = 0$

$\Rightarrow \sin(x) = -\cos(x)$ separate and square both sides

$\Rightarrow \sin^2(x) = \cos^2(x)$

$\Rightarrow$ ANY time you square both sides of any equation you MUST CHECK all of the solutions obtained.

$\Rightarrow \sin^2(x) = 1 - \sin^2(x)$ ← (use identity)

$\Rightarrow 2\sin^2(x) = 1$

$\Rightarrow \sin^2(x) = \frac{1}{2}$

$\Rightarrow \sin(x) = \pm \frac{1}{\sqrt{2}}$

Quad check → all four

$x_R = \pi/4 \Rightarrow \pi/4, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

These Must be checked in an equation BEFORE BOTH SIDES WERE SQUARED → I'll use $\sin(x) + \cos(x) = 0$

So $\left\{0, \pi, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

CHK

$x = \frac{\pi}{4}$

$\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2} \neq 0$

NOT A SOL'N

$x = \frac{3\pi}{4}$

$\sin\left(\frac{3\pi}{4}\right) + \cos\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{2}}{2}\right) = 0$ ✓

$x = 5\pi$

$\sin\left(\frac{5\pi}{4}\right) + \cos\left(\frac{5\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right) = -\sqrt{2} \neq 0$

NOT A SOL'N

$x = \frac{7\pi}{4}$

$\sin\left(\frac{7\pi}{4}\right) + \cos\left(\frac{7\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} = 0$ ✓