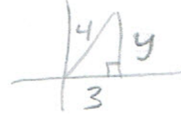
Trigonometry Homework Solutions

Sec 5.1 pg 200

#11. $\cos(\theta) = \frac{3}{4}$ in QI



$\frac{x}{r} = \frac{3}{4} \rightarrow \begin{matrix} x=3 \\ r=4 \end{matrix} \Rightarrow (3)^2 + y^2 = (4)^2$

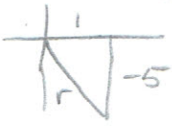
$\Rightarrow y^2 = 16 - 9$

$\Rightarrow y = \pm\sqrt{7}$

$\Rightarrow$ since we are in QI choose $y = +\sqrt{7}$

$\rightarrow$ so $\sin(\theta) = \frac{y}{r} = \boxed{\frac{\sqrt{7}}{4}}$

13. $\cot(\theta) = -\frac{1}{5}$ in QIIII



$\frac{x}{y} = -\frac{1}{5}$ in QIIII $\begin{matrix} x>0 \\ y<0 \end{matrix}$

so $-\frac{1}{5} \Rightarrow \frac{1}{-5}$

$\Rightarrow (1)^2 + (-5)^2 = r^2$

$\Rightarrow r^2 = 26$

$\Rightarrow r = \pm\sqrt{26}$

$\rightarrow$ since r is always positive choose $r = \sqrt{26}$

so

$\Rightarrow \sin(\theta) = \frac{y}{r} = \boxed{\frac{-5}{\sqrt{26}}}$

NO NEED TO RE-WRITE THIS!

15. $\cos(-\theta) = \frac{\sqrt{5}}{3}$ & $\tan(\theta) < 0$

Note that

$\cos(-\theta) = \cos(\theta)$

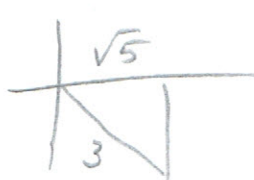
so: $\cos(\theta) = \frac{\sqrt{5}}{3}$

since $\cos(\theta) > 0$ (QI or QIIII)

and $\tan(\theta) < 0$ (QII or QIIII)

we must have

θ in QIIII



so $\frac{x}{r} = \frac{\sqrt{5}}{3}$

so $(\sqrt{5})^2 + y^2 = (3)^2$

$\Rightarrow y^2 = 9 - 5$

$\Rightarrow y = \pm\sqrt{4} = \pm 2$

in QIIII $\Rightarrow y = -2$

so $\sin(\theta) = \frac{y}{r} = \boxed{\frac{-2}{3}}$

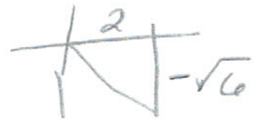
(17). $\tan(\theta) = -\frac{\sqrt{6}}{2}$ $\cos(\theta) > 0$

since $\tan(\theta) < 0$ (QII or QIIII)

<u>and</u> $\cos(\theta) > 0$ (QI or QIIII)

we must have θ in QIIII

and $x > 0$ $y < 0$



so $-\frac{\sqrt{6}}{2} \rightarrow \frac{-\sqrt{6}}{2} = \frac{y}{x}$

so

$(2)^2 + (-\sqrt{6})^2 = r^2$

$\Rightarrow r^2 = 10$

$\Rightarrow r = \sqrt{10}$ (we must have $r > 0$)

so

$\sin(\theta) = \frac{y}{r} = \boxed{\frac{-\sqrt{6}}{\sqrt{10}}}$.

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(19). $\sec(\theta) = \frac{11}{4}$ $\cot(\theta) < 0$

since $\sec(\theta) > 0$ (QI or QIIII)

<u>and</u> $\cot(\theta) < 0$ (QII or QIIII)

we must have θ in QIIII

<u>So</u>: $\sec(\theta) = \frac{r}{x} = \frac{11}{4}$

so $(4)^2 + y^2 = (11)^2$

$\Rightarrow y^2 = 121 - 16$

$\Rightarrow y = \pm\sqrt{105}$

in QIIII so $\Rightarrow y = -\sqrt{105}$

<u>so</u>

$\sin(\theta) = \boxed{\frac{-\sqrt{105}}{11}}$

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(21). $\csc(\theta) = -\frac{9}{4}$

$\Rightarrow \sin(\theta) = \frac{1}{\csc(\theta)} = \frac{1}{-\frac{9}{4}} = \boxed{-\frac{4}{9}}$

(#53). $\cot(\theta)\sin(\theta) = \frac{\cos(\theta)}{\sin(\theta)} \cdot \sin(\theta)$

$= \frac{\sin^2(\theta)}{\cos(\theta)}$

$= \frac{1 - \cos^2(\theta)}{\cos(\theta)}$

split the numerator

$= \frac{1}{\cos(\theta)} - \frac{\cos^2(\theta)}{\cos(\theta)}$

$= \boxed{\sec(\theta) - \cos(\theta)}$

#55. $\sec(\theta)\cot(\theta)\sin(\theta) =$

$$= \frac{1}{\cos(\theta)} \cdot \frac{\sin(\theta)}{\cos(\theta)} \cdot \frac{\sin(\theta)}{1}$$

$$= \frac{\sin^2(\theta)}{\cos^2(\theta)} = \frac{1-\cos^2(\theta)}{\cos^2(\theta)}$$

$$= \frac{1}{\cos^2(\theta)} - \frac{\cos^2(\theta)}{\cos^2(\theta)}$$

$$= \sec^2(\theta) - 1$$

$$= \boxed{\tan^2(\theta)}$$

#57. $\cos(\theta) \cdot \csc(\theta) = \cos(\theta) \cdot \frac{1}{\sin(\theta)}$

$$= \frac{\cos(\theta)}{\sin(\theta)} = \boxed{\tan(\theta)}$$

59. $\sin^2(\theta)(\csc^2(\theta) - 1) = \sin^2(\theta) \cdot \csc^2(\theta) - \sin^2(\theta)$

DISTRIBUTE

$$= \frac{\sin^2(\theta)}{\sin^2(\theta)} - \sin^2(\theta)$$

$$= 1 - \sin^2(\theta)$$

$$= \boxed{\cos^2(\theta)}$$

F O I L

61. $(1 - \cos(\theta))(1 + \sec(\theta)) =$

$$= 1 + \sec(\theta) - \cos(\theta) - \cos(\theta)\sec(\theta)$$

$$= 1 + \frac{1}{\cos(\theta)} - \cos(\theta) - \frac{\cos(\theta)}{\cos(\theta)}$$

$$= \cancel{1} + \frac{1}{\cos(\theta)} - \frac{\cos^2(\theta)}{\cos(\theta)} - \cancel{1}$$

get common denominator

$$= \frac{1-\cos^2(\theta)}{\cos(\theta)}$$

(cont)

#61 cont.

$$= \frac{1-\cos^2(\theta)}{\cos(\theta)}$$

$$= \frac{\sin^2(\theta)}{\cos(\theta)}$$

$$= \frac{\sin(\theta)}{\cos(\theta)} \cdot \frac{\sin(\theta)}{1} = \boxed{\cot(\theta) \cdot \sin(\theta)}$$

#67. $\sec(\theta) - \cos(\theta) = \frac{1}{\cos(\theta)} - \frac{\cos(\theta)}{1}$

get a common denom.

$$= \frac{1}{\cos(\theta)} - \frac{\cos^2(\theta)}{\cos(\theta)}$$

$$= \frac{1-\cos^2(\theta)}{\cos(\theta)}$$

$$= \frac{\sin^2(\theta)}{\cos(\theta)} = \frac{\sin(\theta)}{\cos(\theta)} \cdot \frac{\sin(\theta)}{1}$$

$$= \boxed{\cot(\theta) \cdot \sin(\theta)}$$

F O I L

#69 $(\sec(\theta) + \csc(\theta))(\cos(\theta) - \sin(\theta)) =$

$$= \sec(\theta) \cdot \cos(\theta) - \sec(\theta) \cdot \sin(\theta) + \csc(\theta) \cdot \cos(\theta) - \csc(\theta)\sin(\theta)$$

$$= \frac{1}{\cos(\theta)} \cdot \frac{\cos(\theta)}{1} - \frac{1}{\cos(\theta)} \cdot \frac{\sin(\theta)}{1} + \frac{1}{\sin(\theta)} \cdot \frac{\cos(\theta)}{1} - \frac{1}{\sin(\theta)} \cdot \frac{\sin(\theta)}{1}$$

$$= \cancel{1} - \frac{\sin(\theta)}{\cos(\theta)} + \frac{\cos(\theta)}{\sin(\theta)} - \cancel{1}$$

$$= \boxed{-\cot(\theta) + \tan(\theta)} \text{ or } \boxed{\tan(\theta) - \cot(\theta)}$$

#71. $\sin(\theta)\left(\csc(\theta) - \sin(\theta)\right) =$

$= \sin(\theta)\cdot\csc(\theta) - \sin^2(\theta)$

$= \frac{\sin(\theta)}{1}\cdot\frac{1}{\sin(\theta)} - \sin^2(\theta)$

$= \frac{\sin(\theta)}{\sin(\theta)} - \sin^2(\theta)$

$= 1 - \sin^2(\theta)$

$= \boxed{\cos^2(\theta)}$ ←