TRIGONOMETRY Homework solutions

Sec 5.2 pg 209

#45 $\dfrac{\cot(\theta)}{\csc(\theta)} = \cos(\theta)$

START WITH THE MORE COMPLICATED SIDE.

$$\frac{\cot(\theta)}{\csc(\theta)} = \frac{\frac{\cos(\theta)}{\sin(\theta)}}{\frac{1}{\sin(\theta)}}$$

simplify the complex fraction
=> multiply by the reciprocal

$$= \frac{\cos(\theta)}{\sin(\theta)} \cdot \frac{\sin(\theta)}{1}$$

$$= \cos(\theta) \leftarrow$$

→ OBTAIN THE "OTHER SIDE" OF THE ORIGINAL

#47. $\dfrac{1-\sin^2(\beta)}{\cos(\beta)} = \cos(\beta)$ ← START WITH MORE COMPLICATED SIDE

$$\frac{1-\sin^2(\beta)}{\cos(\beta)} = \frac{\cos^2(\beta)}{\cos(\beta)}$$

$$\Rightarrow \begin{cases} \sin^2(\beta) + \cos^2(\beta) = 1 \\ \sin^2(\beta) = 1 - \cos^2(\beta) \\ \cos^2(\beta) = 1 - \sin^2(\beta) \end{cases}$$

you must know these!!

$$= \cos(\beta) \leftarrow$$

#49. $\cos^2(\theta)\left(\tan^2(\theta)+1\right) = 1$

$$\cos^2(\theta)\left(\tan^2(\theta)+1\right) = \cos^2(\theta)\tan^2(\theta) + \cos^2(\theta)$$

$$= \frac{\cancel{\cos^2(\theta)}}{1} \cdot \frac{\sin^2(\theta)}{\cancel{\cos^2(\theta)}} + \cos^2(\theta)$$

$$= \sin^2(\theta) + \cos^2(\theta)$$

$$= 1 \leftarrow$$

#51. $\cot(\theta)+\tan(\theta)=\sec(\theta)\csc(\theta)$

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start with the "more complicated" side. With fractions add/subtract is usually "more complicated" than multiply/divide.

$$\cot(\theta)+\tan(\theta)=\frac{\cos(\theta)}{\sin(\theta)}+\frac{\sin(\theta)}{\cos(\theta)}$$

- Now get a common denominator

$$=\frac{\cos(\theta)\cdot\cos(\theta)}{\sin(\theta)\cdot\cos(\theta)}+\frac{\sin(\theta)\cdot\sin(\theta)}{\sin(\theta)\cdot\cos(\theta)}$$

$$=\frac{\cos^2(\theta)+\sin^2(\theta)}{\sin(\theta)\cdot\cos(\theta)}$$

$$=\frac{1}{\sin(\theta)\cdot\cos(\theta)}$$

$$=\frac{1}{\sin(\theta)}\cdot\frac{1}{\cos(\theta)}=\csc(\theta)\cdot\sec(\theta)$$

$$=\boxed{\sec(\theta)\csc(\theta)}$$

ALWAYS START WITH ONE SIDE AND CHANGE IT UNTIL YOU GET THE OTHER SIDE.

NEVER, NEVER, NEVER, EVER, NEVER ~~EVER!~~ WORK BOTH SIDES SIMULTANEOUSLY.

THE FOLLOWING IS INVALID AND GETS NO CREDIT

⇓

$$\cot(\theta)+\tan(\theta)=\sec(\theta)\csc(\theta)$$

$$\frac{\cos(\theta)}{\sin(\theta)}+\frac{\sin(\theta)}{\cos(\theta)}=\frac{1}{\cos(\theta)}\cdot\frac{1}{\sin(\theta)}$$

$$\frac{\cos^2+\sin^2(\theta)}{\sin(\theta)\cdot\cos(\theta)}=\frac{1}{\sin(\theta)\cos(\theta)}$$

$$\frac{1}{\sin(\theta)\cdot\cos(\theta)}=\frac{1}{\sin(\theta)\cos(\theta)}$$

This Looks correct BUT IS NOT Logically Valid

This (or anything like it) gets NO CREDIT!

You MUST start with one side and work it until you get the other side.

#53 $\frac{\cos(\alpha)}{\sec(\alpha)} + \frac{\sin(\alpha)}{\csc(\alpha)} = \sec^2(\alpha) - \tan^2(\alpha)$ pg three

NOTE: IT IS A VALID TECHNIQUE TO Start with one $\underbrace{\text{side}}_{}$ and simplify it. (FROM SCRATCH)

THEN start with the other side (AGAIN, FROM SCRATCH) and simplify it.

If BOTH SIDES INDEPENDENTLY simplify to the same thing, then that is a valid proof.

This is the technique here because BOTH sides are very complicated:

LEFT SIDE

$$\frac{\cos(\alpha)}{\sec(\alpha)} + \frac{\sin(\alpha)}{\csc(\alpha)} = \frac{\cos(\alpha)}{\frac{1}{\cos(\alpha)}} + \frac{\sin(\alpha)}{\frac{1}{\sin(\alpha)}}$$

$$= \frac{\cos(\alpha)}{1} \cdot \frac{\cos(\alpha)}{1} + \frac{\sin(\alpha)}{1} \cdot \frac{\sin(\alpha)}{1}$$

$$= \cos^2(\alpha) + \sin^2(\alpha)$$

pythagorean rule $\Longrightarrow$ $= 1$

RIGHT SIDE

$$\sec^2(\alpha) - \tan^2(\alpha) =$$

$$= \frac{1}{\cos^2(\alpha)} - \frac{\sin^2(\alpha)}{\cos^2(\alpha)}$$

denominators are already common.

$$= \frac{1 - \sin^2(\alpha)}{\cos^2(\alpha)}$$

$$= \frac{\cos^2(\alpha)}{\cos^2(\alpha)}$$

$$= 1$$

Thus the statement is true. ✓

(57) $\dfrac{1-\cos(x)}{1+\cos(x)} = (\cot(x) - \csc(x))^2$

LEFT SIDE:

multiply num & denom by conjugate of the denom.

$$\frac{(1-\cos(x))}{(1+\cos(x))} = \frac{(1-\cos(x))}{(1+\cos(x))} \cdot \frac{(1-\cos(x))}{(1-\cos(x))}$$

$$= \frac{1+2\cos(x)+\cos^2(x)}{1-\cos^2(x)}$$

$$= \frac{1+2\cos(x)+\cos^2(x)}{\sin^2(x)}$$ ← SPLIT THE NUMERATOR

$$= \frac{1}{\sin^2(x)} + \frac{2\cos(x)}{\sin^2(x)} + \frac{\cos^2(x)}{\sin^2(x)}$$ ✓

RIGHT SIDE

$$(\cot(x)-\csc(x))^2 = (\cot(x)-\csc(x))(\cot(x)-\csc(x))$$ FOIL

$$= \cot^2(x) - 2\cot(x)\csc(x) + \csc^2(x)$$

$$= \frac{\cos^2(x)}{\sin^2(x)} - \frac{2\cdot\cos(x)}{\sin(x)} \cdot \frac{1}{\sin(x)} + \frac{1}{\sin^2(x)}$$

$$= \frac{1}{\sin^2(x)} - \frac{2\cos(x)}{\sin^2(x)} + \frac{\cos^2(x)}{\sin^2(x)}$$ ✓

MATCH!

Thus the statement is Valid.

(59) $\dfrac{\cos(\theta)+1}{\tan^2(\theta)} = \dfrac{\cos(\theta)}{\sec(\theta)-1}$

$$\frac{\cos(\theta)}{(\sec(\theta)-1)} = \frac{\cos(\theta)\cdot(\sec(\theta)+1)}{(\sec(\theta)-1)(\sec(\theta)+1)} \leftarrow \text{Conjugate of denom!}$$

$$= \frac{\cos(\theta)\cdot\sec(\theta)+\cos(\theta)}{\sec^2(\theta)-1} \qquad \begin{cases}\tan^2(\theta)+1=\sec^2(\theta)\\ \tan^2(\theta)=\sec^2(\theta)-1\end{cases}$$

pythagorean rules!

$$= \frac{\frac{\cos(\theta)}{1}\cdot\frac{1}{\cos(\theta)}+\cos(\theta)}{\tan^2(\theta)} = \frac{1+\cos(\theta)}{\tan^2(\theta)} \leftarrow$$

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(61) $\dfrac{1}{(1-\sin(\theta))} + \dfrac{1}{(1+\sin(\theta))} = 2\sec^2(\theta)$

$$\frac{1}{(1-\sin(\theta))} + \frac{1}{(1+\sin(\theta))} = \frac{(1+\sin(\theta)) + (1-\sin(\theta))}{(1-\sin(\theta))\cdot(1+\sin(\theta))}$$

common denominator!

$$= \frac{2+\cancel{\sin(\theta)}-\cancel{\sin(\theta)}}{1-\sin^2(\theta)}$$

$$= \frac{2}{\cos^2(\theta)}$$

$$= 2\sec^2(\theta) \leftarrow$$

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(63) $$\frac{(\cot(\alpha)+1)}{(\cot(\alpha)-1)} = \frac{(1+\tan(\alpha))}{(1-\tan(\alpha))}$$

Left side

$$\frac{(\cot(\alpha)+1)}{(\cot(\alpha)-1)} = \frac{\frac{\cos(\alpha)}{\sin(\alpha)} + \frac{\sin(\alpha)}{\sin(\alpha)}}{\frac{\cos(\alpha)}{\sin(\alpha)} - \frac{\sin(\alpha)}{\sin(\alpha)}}$$

$$= \frac{\frac{\cos(\alpha)+\sin(\alpha)}{\sin(\alpha)}}{\frac{\cos(\alpha)-\sin(\alpha)}{\sin(\alpha)}}$$

$$= \frac{\cos(\alpha)+\sin(\alpha)}{\cancel{\sin(\alpha)}} \cdot \frac{\cancel{\sin(\alpha)}}{\cos(\alpha)-\sin(\alpha)}$$

$$= \frac{\cos(\alpha)+\sin(\alpha)}{\cos(\alpha)-\sin(\alpha)} \checkmark$$

SAME!

RIGHT SIDE:

$$\frac{1+\tan(\alpha)}{1-\tan(\alpha)} = \frac{\frac{\cos(\alpha)}{\cos(\alpha)} + \frac{\sin(\alpha)}{\cos(\alpha)}}{\frac{\cos(\alpha)}{\cos(\alpha)} - \frac{\sin(\alpha)}{\cos(\alpha)}}$$

$$= \frac{\frac{\cos(\alpha)+\sin(\alpha)}{\cos(\alpha)}}{\frac{\cos(\alpha)-\sin(\alpha)}{\cos(\alpha)}}$$

$$= \frac{\cos(\alpha)+\sin(\alpha)}{\cancel{\cos(\alpha)}} \cdot \frac{\cancel{\cos(\alpha)}}{\cos(\alpha)-\sin(\alpha)}$$

$$= \frac{\cos(\alpha)+\sin(\alpha)}{\cos(\alpha)-\sin(\alpha)} \checkmark$$

Thus the statement is Valid