Solutions Exam I

SOLUTIONS EXAM II pg 1 of 2

I.

① D $f(x) = \sec(5x) \xrightarrow{\text{chain rule}} f'(x) = \sec(5x)\tan(5x)(5)$

② B $f(x) = -3(x+2)^2 + 20x \xrightarrow{\text{chain rule}} f'(x) = -3(2)(x+2)^1(1) + 20$

so $f'(2) = (-3)(2)(2+2)(1) + 20$

$= (-6)(4) + 20$

$= -4 \Rightarrow m = -4$

③ D $2x^4 + 2x^2y = 4y^2 \xrightarrow{\text{IMPLICIT DIFF.}} 8x^3 + 2x^2\frac{dy}{dx} + y(4x) = 8y\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{-4xy - 8x^3}{2x^2 - 8y}$ so for $\left[\frac{dy}{dx}\right]_{x=1}$

Not enough information is given (y is also needed)

④ A $f(x) = \cos(x) - \tan(x) \Rightarrow f'(x) = -\sin(x) - \sec^2(x)$

$\Rightarrow f''(x) = -\cos(x) - 2\sec(x)(\sec(x)\tan(x))$

⑤ D $x^3 + y^3 = 24y \Rightarrow 3x^2 + 3y^2\frac{dy}{dx} = 24\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{-3x^2}{3y^2 - 24}$ so for $\left[\frac{dy}{dx}\right]_{x=1}$ y is also needed so NOT ENOUGH INFO IS GIVEN.

⑥ C A requires product rule but not the chain rule NOT A

B is a basic form so NOT B

D requires quotient rule so NOT D

$f(x) = \sqrt{\csc(x)} = (\csc(x))^{1/2}$ requires chain rule pg 2 ~~w0~~

7. C $\tan(y) + x^2 = 4x - 3 \Rightarrow \sec^2(y)\frac{dy}{dx} + 2x = 4 \Rightarrow \frac{dy}{dx} = \frac{4-2x}{\sec^2(y)}$

so $\left[\frac{dy}{dx}\right]_{(1,0)} = \frac{4-2(1)}{\sec^2(0)} = \frac{2}{1} = 2$

so $m = 2 \Rightarrow y - 0 = 2(x-1) \Rightarrow \boxed{y = 2x - 2}$

8. B $f(x) = \frac{(2x-1)}{(x^2+x)} \Rightarrow f'(x) = \frac{(x^2+x)(2)-(2x-1)(2x+1)}{(x^2+x)^2}$ ←

9. C $\tan(x+y) = 5x \Rightarrow \sec^2(x+y)\left(1 + \frac{dy}{dx}\right) = 5$

10. A $\Rightarrow \frac{dy}{dx} = \frac{5}{\sec^2(x+y)} - 1 \Rightarrow \left[\frac{dy}{dx}\right]_{(0,0)} = \frac{5}{\sec^2(0+0)} = 5 -$

11. D $V = \frac{1}{3}\pi r^2 h$ Find $\frac{dr}{dt}$

differential count:

given $\frac{dV}{dt} = 24\pi$

$r = 4\text{cm}$

$\Rightarrow \frac{dV}{dt}$ ← $\frac{dr}{dt}$ ← $\frac{dh}{dt}$ NO

we need to eliminate h, but we do not have a value for V to do so. ⇒ NOT ENOUGH INFO IS GIVEN ←

12. I discarded this item.

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(13). $f(x) = x^4 \sec(x)$ (product rule)

$\Rightarrow f'(x) = \boxed{x^4 \sec(x)\tan(x) + \sec(x)(4x^3)}$

(14). $f(x) = (-x^{-5} - x^{-2})^{-3}$ (chain rule)

$\Rightarrow f'(x) = \boxed{(-3)(-x^{-5}-x^{-2})^{-4}(5x^{-6}+2x^{-3})}$

(15). $f(x) = \dfrac{4x}{x^{1/4} - \tan(x)}$ (quotient rule)

$$\Rightarrow f'(x) = \boxed{\frac{(x^{1/4} - \tan(x))(4) - (4x)(\frac{1}{4}x^{-3/4} - \sec^2(x))}{(x^{1/4} - \tan(x))^2}}$$

(16). $f(x) = \sin^2(x^2) = (\sin(x^2))^2$ (chain rule TWICE)

$\Rightarrow f'(x) = \boxed{2(\sin(x^2))^1(\cos(x^2)(2x))}$

(17) $f(x) = \cos(x)(x^3 + 4x^{-2})$ (product rule)

$\Rightarrow f'(x) = \boxed{\cos(x)(3x^2 - 8x^{-3}) + (x^3 + 4x^{-2})(-\sin(x))}$

18. $f(x) = \frac{x^2+5}{x^5+2}$ (Quotient rule) pg 4our

$\Rightarrow f'(x) = \boxed{\frac{(x^5+2)(2x) - (x^2+5)(5x^4)}{(x^5+2)^2}}$

19. $f(x) = 2x^4 + 3x^{-2} - 12x$

$\Rightarrow \boxed{f'(x) = 8x^3 - 6x^{-3} - 12}$

$\Rightarrow f''(x) = \boxed{24x^2 + 18x^{-4}}$

20. $f(x) = x + \tan(2x)$ (chain rule)

$\Rightarrow f'(x) = 1 + \sec^2(2x)(2) = 1 + 2\sec^2(2x)$

$\Rightarrow f''(x) = \boxed{2(2\sec(2x)(2))}$

21. $x^3 + 5x^2y^2 = 3y$

$\Rightarrow 3x^2 + (5x^2)(2y\frac{dy}{dx}) + (y^2)(10x) = 3\frac{dy}{dx}$

$\Rightarrow 10x^2y\frac{dy}{dx} - 3\frac{dy}{dx} = -3x^2 - 10xy^2$

$\Rightarrow \frac{dy}{dx} = \boxed{\frac{-3x^2 - 10xy^2}{10x^2y - 3}}$

22. $-\cos(x+4y) = 2x$

$\Rightarrow -(-\sin(x+4y))(1 + 4\frac{dy}{dx}) = 2$

$\Rightarrow \sin(x+4y) + 4\sin(x+4y)\frac{dy}{dx} = 2$

$\Rightarrow 4\sin(x+4y)\frac{dy}{dx} = 2 - \sin(x+4y)$

$\Rightarrow \frac{dy}{dx} = \boxed{\frac{2 - \sin(x+4y)}{\sin(x+4y)}}$

(23) $f(x) = x^2 - 5x - 6$

pg 5 ive $\rightarrow$

$$\Rightarrow f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - 5(x+\Delta x) - 6 - (x^2 - 5x - 6)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 - \cancel{5x} - 5\Delta x - \cancel{6} - \cancel{x^2} + \cancel{5x} + \cancel{6}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2 - 5\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x - 5)}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x - 5)$$

$$= 2x + 0 - 5$$

$$= \boxed{2x - 5}$$

(24). Find $\boxed{\frac{dy}{dx} =}$

given:

$\frac{dx}{dt} = 2$

$\frac{dz}{dt} = -1$

$z = 5$

$y = 4$

start with

$x^2 + y^2 = z^2$

Differential check

$\frac{dx}{dt}$ ✓ $\quad \frac{dz}{dt}$ NO

$\frac{dy}{dt}$ ✓ must eliminate y

$\Rightarrow x^2 + (4)^2 = (5^2)$

$\Rightarrow x = 3$

$\Rightarrow 9 + y^2 = z^2$

$\Rightarrow 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$

$\Rightarrow 2(4) \frac{dy}{dt} = 2(5)(-1)$

$\Rightarrow 8 \frac{dy}{dt} = -10$

$\Rightarrow \frac{dy}{dt} = \boxed{-\frac{10}{8}}$

(25) FIND: $\boxed{\frac{dr}{dt} =}$ pg 6ix

GIVEN:

$V = \pi r^2 h$ $\quad \frac{dV}{dt} = -12$

$h = 3r$ ⊛

$r = 4$

$V = \pi r^2 h$

⊛ differential check

$\frac{dV}{dt}$ ✓ $\frac{dr}{dt}$ ✓ $\frac{dh}{dt}$ NO

must eliminate h

we have $h = 3r$ ⊛

so $V = \pi r^2 (3r)$

so $V = \pi 3 r^3$

$\Rightarrow V = 3\pi r^3$

$\Rightarrow \frac{dV}{dt} = 9\pi r^2 \frac{dr}{dt} \Rightarrow -12 = 9\pi (4)^2 \frac{dr}{dt} \Rightarrow \boxed{\frac{dr}{dt} = \frac{-12}{144\pi} \frac{cm}{sec}}$

(26) FIND $\frac{dA}{dt}$ GIVEN $\frac{dV}{dt} = 72\pi$ $\quad V = 36\pi$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

Need value for r

$\Rightarrow 36\pi = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{36\pi}{\frac{4}{3}\pi} = r^3 \Rightarrow 27 = r^3$

$\Rightarrow r = 3$ ✓

so

$72\pi = 4\pi (3)^2 \frac{dr}{dt}$

$\Rightarrow \frac{72\pi}{12\pi} = \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = 6$

so now

$A = 4\pi r^2$

$\Rightarrow \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$\Rightarrow \frac{dA}{dt} = 8\pi (3)(6)$

$= \boxed{144\pi \ cm^2/sec}$