Solutions Exam III

TRIGONOMETRY EXAM III
SOLUTIONS

(I)

(1) C $\cos(\theta)\tan(\theta)\csc(\theta) = \frac{\cos(\theta)}{1}\cdot\frac{\sin(\theta)}{\cos(\theta)}\cdot\frac{1}{\sin(\theta)} = 1$

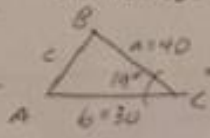
(2) E conjugate of $1-\sin(\theta) = 1+\sin(\theta)$

(3) D $\sec^2(\theta)+\csc(\theta) \neq 1$ so NOT A

$\frac{1}{\cot(\theta)} = \frac{1}{\frac{\cos(\theta)}{\sin(\theta)}} = \frac{\sin(\theta)}{\cos(\theta)} \neq \frac{\cos(\theta)}{\sin(\theta)}$ so NOT B

$\sec^2(\theta)+\csc^2(\theta) \neq \tan^2(\theta)$ so NOT C

(4) A $\sin^2(\theta)\cot(\theta)\cdot\csc(\theta) = \frac{\sin^2(\theta)}{1}\cdot\frac{\cos(\theta)}{\sin(\theta)}\cdot\frac{1}{\sin(\theta)} = \cos(\theta)$

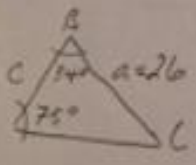
(5) B 

$\Rightarrow c^2 = (40)^2+(30)^2-2(40)(30)\cos(19°)$
≈ 230.755
$c \approx \sqrt{230.755} \approx 15.19$

(6) B Law of sines is true for any Δ so NOT C AND LAW of cosines is true for any Δ so NOT A NOT D

(7) A $\tan(\theta)\sin(\theta)\cot(\theta)\sec(\theta) =$
$= \frac{\sin(\theta)}{\cos(\theta)}\cdot\frac{\sin(\theta)}{1}\cdot\frac{\cos(\theta)}{\sin(\theta)}\cdot\frac{1}{\cos(\theta)} = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$
so A

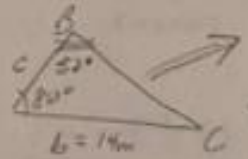
(8) C $\sin^2(\theta)+\cos^2(\theta) = 1 \Rightarrow \sin^2(\theta) = 1-\cos^2(\theta)$
so C

(9) E 

$\angle C = 180° - (34°+75°) = 71°$
Law of Sines (so NOT D)
$\frac{\sin(71°)}{c} = \frac{\sin(75°)}{26} \Rightarrow c = \frac{26\sin(71°)}{\sin(75°)} \approx 25.45$ m

(10). C — NOT A, as Law of Cosines applies
NOT B, as Law of Sines applies
NOT D as Law of Cosines applies
SINCE NO sides are given there is NO UNIQUE TRIANGLE, SO C.

(11). E $\sin(\theta)\cot(\theta)\cos(\theta) = \frac{\sin(\theta)}{1}\cdot\frac{\cos(\theta)}{\sin(\theta)}\cdot\frac{\cos(\theta)}{1} = \cos^2(\theta)$

(12). B 

$\angle C = 180° - (80° + 52°) = 48°$

$\Rightarrow \frac{\sin(48°)}{c} = \frac{\sin(52°)}{14}$

$\Rightarrow c = \frac{14\sin(48°)}{\sin(52°)} \approx 13.2\text{ m}$

(13). $\frac{1+\cot(\theta)}{\cot(\theta)} \cdot \frac{1+\frac{\cos(\theta)}{\sin(\theta)}}{\frac{\cos(\theta)}{\sin(\theta)}} = \frac{\frac{\sin(\theta)}{\sin(\theta)}+\frac{\cos(\theta)}{\sin(\theta)}}{\frac{\cos(\theta)}{\sin(\theta)}} =$

$= \frac{\frac{\sin(\theta)+\cos(\theta)}{\sin(\theta)}}{\frac{\cos(\theta)}{\sin(\theta)}} = \frac{\sin(\theta)+\cos(\theta)}{\sin(\theta)}\cdot\frac{\sin(\theta)}{\cos(\theta)} =$

$= \frac{\sin(\theta)+\cos(\theta)}{\cos(\theta)} = \frac{\sin(\theta)}{\cos(\theta)} + \frac{\cos(\theta)}{\cos(\theta)} = \boxed{\tan(\theta)+1}$

(14). $\frac{\tan(\theta)}{\csc(\theta)} + \cos(\theta) = \frac{\frac{\sin(\theta)}{\cos(\theta)}}{\frac{1}{\sin(\theta)}} + \cos(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \cdot \frac{\sin(\theta)}{1} + \frac{\cos(\theta)}{1} \cdot$

$= \frac{\sin^2(\theta)}{\cos(\theta)} + \frac{\cos(\theta)}{1} \cdot \frac{\sin^2(\theta)}{\cos(\theta)} + \frac{\cos^2(\theta)}{\cos(\theta)} \cdot \frac{\sin^2(\theta) + \cos^2(\theta)}{\cos(\theta)} =$

$= \frac{1}{\cos(\theta)} = \boxed{\sec(\theta)}$

(15). Verify $\frac{1 + \cot^2(\theta)}{\csc(\theta)} = \csc(\theta)$

LONG VERSION:

$\frac{1 + \cot^2(\theta)}{\csc(\theta)} = \frac{1 + \frac{\cos^2(\theta)}{\sin^2(\theta)}}{\frac{1}{\sin(\theta)}} = \frac{\frac{\sin^2(\theta)}{\sin^2(\theta)} + \frac{\cos^2(\theta)}{\sin^2(\theta)}}{\frac{1}{\sin(\theta)}} =$

$= \frac{\sin^2(\theta) + \cos^2(\theta)}{\sin^2(\theta)} \cdot \frac{\sin(\theta)}{1} = \frac{1}{\sin^2(\theta)} \cdot \frac{\sin(\theta)}{1} =$

$= \frac{1}{\sin(\theta)} = \boxed{\csc(\theta)}$ ←

OR

SHORT VERSION: $\frac{1 + \cot^2(\theta)}{\csc(\theta)} = \frac{\csc^2(\theta)}{\csc(\theta)} = \boxed{\csc(\theta)}$ —

=

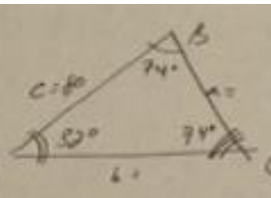
(16.) Verify $\frac{\sec(\theta)+\tan(\theta)}{\cot(\theta)+\cos(\theta)} = \tan(\theta)\sec(\theta)$ pg 4 our

$$\frac{\sec(\theta)+\tan(\theta)}{\cot(\theta)+\cos(\theta)} = \frac{\frac{1}{\cos(\theta)}+\frac{\sin(\theta)}{\cos(\theta)}}{\frac{\cos(\theta)}{\sin(\theta)}+\frac{\cos(\theta)}{1}} = \frac{\frac{1+\sin(\theta)}{\cos(\theta)}}{\frac{\cos(\theta)}{\sin(\theta)}+\frac{\sin(\theta)\cos(\theta)}{\sin(\theta)}} =$$

$$= \frac{\frac{1+\sin(\theta)}{\cos(\theta)}}{\frac{\cos(\theta)+\sin(\theta)\cos(\theta)}{\sin(\theta)}} = \frac{\frac{1+\sin(\theta)}{\cos(\theta)}}{\frac{\cos(\theta)(1+\sin(\theta))}{\sin(\theta)}} =$$

$$= \frac{(1+\sin(\theta))}{\cos(\theta)} \cdot \frac{\sin(\theta)}{\cos(\theta)(1+\sin(\theta))} = \frac{\sin(\theta)}{\cos(\theta)\cdot\cos(\theta)} = \tan(\theta)\sec(\theta)$$

(#17)



FIND: $\angle C = 74°$, $a = 44.1\,m$, $b = 80\,m$

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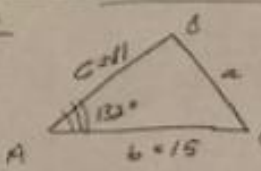
$\Rightarrow$ ASA is Given

$\angle C$: $\angle C = 180° - 32° - 74°$
$= 74°$

a: $\frac{\sin(74°)}{80} = \frac{\sin(32°)}{a}$
$\Rightarrow a\sin(74°) = 80\sin(32°) \Rightarrow a = \frac{80\sin(32°)}{\sin(74°)}$
$= 44.1$

b: $\frac{\sin(74°)}{80} = \frac{\sin(74°)}{b} \Rightarrow b = 80m$

(18)



FIND: $\angle B = 27.9°$, $\angle C = 20.1°$, $a = 23.8\,m$

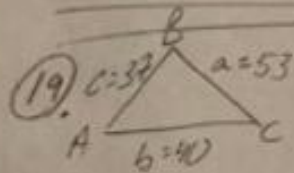
$\Rightarrow$ SAS is Given

a: $a^2 = b^2 + c^2 - 2bc\cos(A)$
$= (15)^2 + (11)^2 - 2(15)(11)\cos(132°)$
$= 225 + 121 - (-220.813\ldots)$
$= 566.813\ldots$
$\Rightarrow a = \sqrt{566.813\ldots} = 23.807\ldots \Rightarrow 23.8$

$\angle C$: $\frac{\sin(C)}{11} = \frac{\sin(132°)}{23.8}$
$\Rightarrow 23.8\sin(C) = 11\sin(132°)$
$\Rightarrow \sin(C) = \frac{11\sin(132°)}{23.8}$
$= .3433\ldots$
$\angle C = \sin^{-1}(.3433\ldots) = 20.081\ldots°$
$\Rightarrow \angle C = 20.1°$

$\angle B$: $\angle B = 180° - 132° - 20.1°$
$= 27.9°$

(19)



FIND: $\angle A = 86.9°$, $\angle B = 48.9°$, $\angle C = 44.2°$

$\angle B$: $\frac{\sin(B)}{40} = \frac{\sin(86.9)}{53}$
$\Rightarrow \sin(B) = \frac{40\cdot\sin(86.9)}{53}$
$\Rightarrow \angle B = \sin^{-1}(.7536\ldots)$
$= 48.9°$

$\angle C$:
$\angle C = 180° - 86.9° - 48.9°$
$= 44.2°$

$\angle A$: $a^2 = b^2 + c^2 - 2bc\cos(A)$
$\Rightarrow (53)^2 = (40)^2 + (37)^2 - 2(40)(37)\cos(A)$
$\Rightarrow 2809 = 2969 - 2960\cos(A)$
$\Rightarrow \frac{2809 - 2969}{-2960} = \cos(A) \Rightarrow \cos(A) = .0540\ldots$

50 $\angle A = \cos^{-1}(.0540)$
$= 86.9°$