Basic Forms from Calculus I

1. Definition of a Limit:

$$\lim_{x\to a} f(x) = L \Leftrightarrow \forall \varepsilon > 0, \exists \delta \text{ such that } 0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

OR

$\lim_{x\to a} f(x) = L$ if and only if:

for all $\varepsilon > 0$ there exists δ such that $0 < |x-a| < \delta$ implies $|f(x) - L| < \varepsilon$

2. $\lim_{x\to a} k = k$

3. $\lim_{x\to a} x = a$

4. $\lim_{x\to a} x^n = a^n$

5. $\lim_{x\to a} [kf(x)] = k \lim_{x\to a} [f(x)]$

6. $\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} [f(x)] \pm \lim_{x\to a} [g(x)]$

7. $\lim_{x\to a} [f(x) \cdot g(x)] = \lim_{x\to a} [f(x)] \cdot \lim_{x\to a} [g(x)]$

8. $\lim_{x\to a} \left[[f(x)]^n \right] = \left[\lim_{x\to a} [f(x)] \right]^n$

9. $\lim_{x\to a} \left[\dfrac{f(x)}{g(x)} \right] = \dfrac{\lim_{x\to a} [f(x)]}{\lim_{x\to a} [g(x)]}$ $\quad \lim_{x\to a} [g(x)] \neq 0$

10. $\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x) \right)$

11. $\lim_{x\to 0} \dfrac{\sin(x)}{x} = 1$

12. $\lim_{x\to 0} \dfrac{1 - \cos(x)}{x} = 0$

13. Definition of Continuity:

$f(x)$ is continuous at $x = a$ if and only if:

A. $f(a)$ is defined

B. $\lim_{x\to a} f(x)$ exists

C. $\lim_{x\to a} f(x) = f(a)$

14. Definition of the Derivative:

$$\frac{d}{dx}[f(x)] = \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

15. $\dfrac{d}{dx}[k] = 0$

16. $\dfrac{d}{dx}\left[u^n\right] = nu^{n-1} \cdot u'$

17. $\dfrac{d}{dx}[k\,u] = k\,u'$

18. $\dfrac{d}{dx}[u \pm v] = u' \pm v'$

19. $\dfrac{d}{dx}[u\,v] = u\,v' + v\,u'$

20. $\dfrac{d}{dx}\left[\dfrac{u}{v}\right] = \dfrac{v\,u' - u\,v'}{v^2}$

21. $\dfrac{d}{dx}[f(u)] = f'(u) \cdot u'$

22. $\dfrac{d}{dx}\left[e^u\right] = e^u \cdot u'$

23. $\dfrac{d}{dx}[\ln(u)] = \dfrac{1}{u} \cdot u'$

24. $\frac{d}{dx}\left[\sin(u)\right]=\cos(u)\cdot u'$

25. $\frac{d}{dx}\left[\csc(u)\right]=-\csc(u)\cot(u)\cdot u'$

26. $\frac{d}{dx}\left[\cos(u)\right]=-\sin(u)\cdot u'$

27. $\frac{d}{dx}\left[\sec(u)\right]=\sec(u)\tan(u)\cdot u'$

28. $\frac{d}{dx}\left[\tan(u)\right]=\sec^2(u)\cdot u'$

29. $\frac{d}{dx}\left[\cot(u)\right]=-\csc^2(u)\cdot u'$

30. $\frac{d}{dx}\left[\sin^{-1}(u)\right]=\frac{u'}{\sqrt{1-u^2}}$

31. $\frac{d}{dx}\left[\tan^{-1}(u)\right]=\frac{u'}{1+u^2}$

32. $\frac{d}{dx}\left[\sec^{-1}(u)\right]=\frac{u'}{|u|\sqrt{u^2-1}}$

33. $\int k\ du=ku+c$

34. $\int u^n\ du=\frac{u^{n+1}}{n+1}+c \quad n\neq -1$

35. $\int k\ f(u)\ du=k\int f(u)\ du$

36. $\int\left[f(u)\pm g(u)\right]du=\int f(u)du\pm\int g(u)du$

37. $\int e^u\ du=e^u+c$

38. $\int\frac{1}{u}\ du=\ln|u|+c$

39. $\int u^{-1}\ du=\ln|u|+c$

40. $\int\sin(u)\ du=-\cos(u)+c$

41. $\int\csc(u)\cot(u)\ du=-\csc(u)+c$

42. $\int\cos(u)\ du=\sin(u)+c$

43. $\int\sec(u)\tan(u)\ du=\sec(u)+c$

44. $\int\tan(u)\ du=\ln|\sec(u)|+c$

45. $\int\cot(u)\,du=\ln|\sin(u)|+c$

46. $\int\sec^2(u)\ du=\tan(u)+c$

47. $\int\csc^2(u)\ du=-\cot(u)+c$

48. $\int\sec(u)\ du=\ln|\sec(u)+\tan(u)|+c$

49. $\int f(au+b)\ du=\frac{1}{a}F(au+b)+c$

50. $\int\frac{du}{\sqrt{a^2-u^2}}=\sin^{-1}\left(\frac{u}{a}\right)+c$

51. $\int\frac{1}{a^2+u^2}du=\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C$

52. $\int\frac{du}{u\sqrt{u^2-a^2}}=\frac{1}{a}\sec^{-1}\left(\frac{|u|}{a}\right)+c$