Extra Continuity Homework Solutions

1) $f(x) = 3x^4 - 5x^2 + 6$ at $x = 2$

i) $f(2) = 3(2)^4 - 5(2)^2 + 6 = 34$ ✓

ii) $\lim_{x \to 2} f(x) = \lim_{x \to 2} (3x^4 - 5x^2 + 6) = 34$ ✓

iii) $\lim_{x \to 2} f(x) = 34 = f(2)$ ✓

so $f(x)$ is continuous at $x = 2$

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2) $f(x) = \dfrac{6x}{x^2 + 2x - 15}$ at $x = -5$

i) $f(-5) = \dfrac{6(-5)}{(-5)^2 + 2(-5) - 15} = \dfrac{-30}{0}$ = undefined ✗

so $f(x)$ is not continuous at $x = -5$

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3) $f(x) = \begin{cases} x^2 - 3x & x \neq 5 \\ x + 4 & x = 5 \end{cases}$ at $x = 5$

i) $f(5) = (5) + 4 = 9$ ✓

ii) $\lim_{x \to 5} f(x) \overset{\text{TOP}}{=} \lim_{x \to 5} (x^2 - 3x) = (5)^2 - 3(5) = 10$ ✓

iii) $\lim_{x \to 5} f(x) \neq f(5)$ ✗

so $f(x)$ is not continuous at $x = 5$

4) $f(x) = \begin{cases} 10 - x^2 & x \leq 3 \\ 2x - 5 & x > 3 \end{cases}$ at $x = -2$

i) $f(-2) \overset{\text{TOP}}{=} 10 - (-2)^2 = 10 - 4 = 6$ ✓

ii) $\lim_{x \to -2} f(x) \overset{\text{TOP}}{=} \lim_{x \to -2} (10 - x^2) = 6$ ✓

iii) $\lim_{x \to -2} f(x) = 6 = f(-2)$ ✓

so $f(x)$ is continous at $x = -2$

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5) $f(x) = \begin{cases} 3 - \cos(x) & x > \pi \\ \tan(x) + 3 & x \leq \pi \end{cases}$ at $x = \pi$

i) $f(\pi) \overset{\text{BOT}}{=} \tan(\pi) + 3 = 0 + 3 = 3$ ✓

ii) $\lim_{x \to \pi} f(x) = \text{DNE}$

a) $\lim_{x \to \pi^-} f(x) \overset{\text{BOT}}{=} \lim_{x \to \pi^-} (\tan(\pi) + 3) = 0 + 3 = 3$

b) $\lim_{x \to \pi^+} f(x) \overset{\text{TOP}}{=} \lim_{x \to \pi^+} (3 - \cos(x)) = 3 - \cos(\pi)$
$= 3 - (-1)$
$= 3 + 1$
$= 4$

so $f(x)$ is not continuous at $x = \pi$

6) $f(x) = \frac{x^3 - 4x}{x^2 - x}$ at $x = 0$

i) $f(0) = \frac{(0)^3 - 4(0)}{(0)^2 - (0)} = \frac{0}{0}$ undefined ✗

so $f(x)$ is not continuous at $x = 0$

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7) $f(x) = \sin(2x) + \tan(x) + 1$ at $x = \frac{\pi}{4}$

i) $f(\frac{\pi}{4}) = \sin(2 \cdot \pi/4) + \tan(\frac{\pi}{4}) + 1$

$= \sin(\frac{\pi}{2}) + \tan(\frac{\pi}{4}) + 1$

$= 1 + 1 + 1$

$= 3$ ✓

ii) $\lim_{x \to \pi/4} f(x) = \lim_{x \to \pi/4} [\sin(2x) + \tan(x) + 1]$ ⇒

$= \sin(2 \cdot \pi/4) + \tan(\pi/4) + 1$

$= 3$ ✓

iii) $\lim_{x \to \pi/4} f(x) = 3 = f(\pi/4)$

so $f(x)$ is continuous at $x = \pi/4$