SOLUTIONS

1. Prove $\lim_{x\to 2}(4x-3)=5$

(i) Let $\epsilon>0$

(ii) search for δ:

we want

$|f(x)-L|<\epsilon$

$\Rightarrow |(4x-3)-5|<\epsilon$

$\Rightarrow |4x-8|<\epsilon$

$\Rightarrow |4(x-2)|<\epsilon$

$\Rightarrow |4|\,|x-2|<\epsilon$

$\Rightarrow |x-2|<\dfrac{\epsilon}{|4|}$

Choose $\delta=\dfrac{\epsilon}{|4|}$

(iii) Verify this δ:

$0<|x-a|<\delta \Rightarrow |x-2|<\dfrac{\epsilon}{|4|}$

$\Rightarrow |4|\,|x-2|<\epsilon$

$\Rightarrow |4(x-2)|<\epsilon$

$\Rightarrow |4x-8|<\epsilon$

$\Rightarrow |4x-3+3-8|<\epsilon$

$\Rightarrow |(4x-3)-5|<\epsilon$

$\Rightarrow |f(x)-L|<\epsilon$

2. Prove $\lim_{x\to 4}(-6x+17)=-7$

(i) Let $\epsilon>0$

(ii) search for δ:

we want

$|f(x)-L|<\epsilon$

$\Rightarrow |(-6x+17)-(-7)|<\epsilon$

$\Rightarrow |-6x+17+7|<\epsilon$

$\Rightarrow |-6x+24|<\epsilon$ NOTE!

$\Rightarrow |-6(x-4)|<\epsilon$

$\Rightarrow |-6|\,|x-4|<\epsilon$

$\Rightarrow |x-4|<\dfrac{\epsilon}{|-6|}$

choose $\delta=\dfrac{\epsilon}{|-6|}$

(iii) Verify this δ:

$0<|x-a|<\delta \Rightarrow |x-4|<\dfrac{\epsilon}{|-6|}$

$\Rightarrow |-6|\,|x-4|<\epsilon$

$\Rightarrow |-6(x-4)|<\epsilon$

$\Rightarrow |-6x+24|<\epsilon$

$\Rightarrow |-6x+17-17+24|<\epsilon$

$\Rightarrow |(-6x+17)+7|<\epsilon$

$\Rightarrow |(-6x+17)-(-7)|<\epsilon$

$\Rightarrow |f(x)-L|<\epsilon$

Note this MUST be "minus"

③

3) Prove $\lim_{x \to (-5)} (2x+13) = 3$

i) Let $\epsilon > 0$

ii) Search for δ:

we want

$|f(x) - L| < \epsilon$

$\Rightarrow |(2x+13) - 3| < \epsilon$

$\Rightarrow |2x+10| < \epsilon$

$\Rightarrow |2(x+5)| < \epsilon$

$\Rightarrow |2|\,|x+5| < \epsilon$

$\Rightarrow |2|\,|x-(-5)| < \epsilon$

$\Rightarrow |x - (-5)| < \frac{\epsilon}{|2|}$

NOTE: This must be "minus"
and This must be "a"

Choose $\delta = \frac{\epsilon}{|2|}$

iii) Verify this δ:

$0 < |x-a| < \delta \Rightarrow |x-(-5)| < \frac{\epsilon}{|2|}$

$\Rightarrow |x+5| < \frac{\epsilon}{|2|}$

$\Rightarrow |2|\,|x+5| < \epsilon$

$\Rightarrow |2(x+5)| < \epsilon$

$\Rightarrow |2x+10| < \epsilon$

$\Rightarrow |2x+13-13+10| < \epsilon$

$\Rightarrow |(2x+13)-3| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$

④ Prove $\lim_{x \to 0} (6x-4) = -4$ careful!

i) Let $\epsilon > 0$.

ii) search for δ:

we want

$|f(x) - L| < \epsilon$

$\Rightarrow |(6x-4) - (-4)| < \epsilon$

$\Rightarrow |6x-4+4| < \epsilon$

$\Rightarrow |6x| < \epsilon$

$\Rightarrow |6|\,|x| < \epsilon$

*NOTE: we may write this as

$|6|\,|x-0| < \epsilon$

so this is "$|x-a|$"

$\Rightarrow |x-0| < \frac{\epsilon}{|6|}$

choose $\delta = \frac{\epsilon}{|6|}$

iii) Verify this δ:

$0 < |x-a| < \delta \Rightarrow |x-0| < \frac{\epsilon}{|6|}$

$\Rightarrow |6|\,|x| < \epsilon$

$\Rightarrow |6x| < \epsilon$

$\Rightarrow |6x-4+4| < \epsilon$

$\Rightarrow |(6x-4)-(-4)| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$

5. Prove $\lim_{x \to (-7)} (x+3) = -4$

I Let $\epsilon > 0$

II search for δ:

we want

$|f(x) - L| < \epsilon$

$\Rightarrow |(x+3) - (-4)| < \epsilon$

$\Rightarrow |x+3+4| < \epsilon$

$\Rightarrow |x+7| < \epsilon$

$\Rightarrow \underbrace{|x-(-7)|} < \epsilon$

Note: we have created "$|x-a|$" on the left-hand side, so choose δ = "whatever is on the right hand side."

In this case:

choose $\delta = \epsilon$

III Verify this δ:

$0 < |x-a| < \delta \Rightarrow |x-(-7)| < \epsilon$

$\Rightarrow |\underbrace{x+7}| < \epsilon$

Note: we must create $f(x)$ here

$\Rightarrow |x+3-3+7| < \epsilon$

$\Rightarrow |x+3+4| < \epsilon$

$\Rightarrow |(x+3) - (-4)| < \epsilon$

Again this sign must be "minus" and this value must be "L"

$\Rightarrow |f(x) - L| < \epsilon$ ✓

6. prove $\lim_{x \to 4} (-4x+4) = -12$

I Let $\epsilon > 0$

II Search for δ:

we want

$|f(x) - L| < \epsilon$

$\Rightarrow |(-4x+4) - (-12)| < \epsilon$

$\Rightarrow |-4x+4+12| < \epsilon$

$\Rightarrow |-4x+16| < \epsilon$

$\Rightarrow |-4(x-4)| < \epsilon$

$\Rightarrow |-4||x-4| < \epsilon$

$\Rightarrow |x-4| < \frac{\epsilon}{|-4|}$

choose $\delta = \frac{\epsilon}{|-4|}$

III Verify this δ:

$0 < |x-a| < \delta \Rightarrow |x-4| < \frac{\epsilon}{|-4|}$

$\Rightarrow |-4||x-4| < \epsilon$

$\Rightarrow |-4(x-4)| < \epsilon$

$\Rightarrow |-4x+16| < \epsilon$

$\Rightarrow |-4x+4-4+16| < \epsilon$

$\Rightarrow |(-4x+4)+12| < \epsilon$

$\Rightarrow |(-4x+4)-(-12)| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$ ✓

7. Prove $\lim_{x\to a}(mx+b) = (ma+b)$

I Let $\epsilon > 0$

II search for δ:

we want:

$|f(x) - L| < \epsilon$

$\Rightarrow |(mx+b) - (ma+b)| < \epsilon$

Remember, we must create "$|x-a|$" here

$\Rightarrow |mx+b-ma-b| < \epsilon$

$\Rightarrow |mx-ma+\cancel{b}-\cancel{b}| < \epsilon$

$\Rightarrow |mx-ma| < \epsilon$

$\Rightarrow |m(x-a)| < \epsilon$

$\Rightarrow |m||x-a| < \epsilon$

$\Rightarrow |x-a| < \dfrac{\epsilon}{|m|}$

so choose $\delta = \dfrac{\epsilon}{|m|}$

<u>Note</u>: since we don't know if m is positive or negative we <u>MUST</u> retain the absolute value.

III Verify this δ:

$0 < |x-a| < \delta \Rightarrow$

$\Rightarrow |x-a| < \dfrac{\epsilon}{|m|}$

$\Rightarrow |m||x-a| < \epsilon$

$\Rightarrow |m(x-a)| < \epsilon$

$\Rightarrow |mx-ma| < \epsilon$

$\Rightarrow |mx+b-b-ma| < \epsilon$

$\Rightarrow |mx+b-ma-b| < \epsilon$

$\Rightarrow |(mx+b)-(ma+b)| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$