SOLUTIONS PRACTICE EXAM #2wo

CALC II FALL 2016

I

NOTE

I HAVE INCLUDED TWO EXTRA PROBLEMS AT THE END OF SOLUTIONS

BE SURE TO LOOK OVER THEM AS WELL

① $\int \sin^2(x)\,dx = \frac{1}{2}\int [1-\cos(2x)]\,dx$

$= \boxed{\frac{1}{2}\left[x - \frac{1}{2}\sin(2x)\right] + C}$

~~②~~ ③. $\int 4x^2 e^{(2x)}\,dx =$

parts $u = 4x^2$, $du = 8x\,dx$; $v = \frac{1}{2}e^{(2x)}$, $dv = e^{(2x)}\,dx$

$= uv - \int v\,du$

$= (4x^2)\left(\frac{1}{2}e^{(2x)}\right) - \int \frac{1}{2}e^{(2x)}(8x)\,dx$

$= 2x^2 e^{(2x)} - \int 4x e^{(2x)}\,dx$

Parts: $u = 4x$, $du = 4\,dx$; $v = \frac{1}{2}e^{(2x)}$, $dv = e^{(2x)}\,dx$

$= 2x^2 e^{(2x)} - \left[uv - \int v\,du\right]$

$= 2x^2 e^{(2x)} - \left[(4x)\left(\frac{1}{2}e^{(2x)}\right) - \int\left(\frac{1}{2}e^{(2x)}\right)(4\,dx)\right]$

$= 2x^2 e^{(2x)} - 2x e^{(2x)} + \int 2e^{(2x)}\,dx$

$= 2x^2 e^{(2x)} - 2x e^{(2x)} + \frac{2}{2}e^{(2x)} + C$

$= \boxed{2x^2 e^{(2x)} - 2x e^{(2x)} + e^{(2x)} + C}$

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(4) $\int \cos^3(x)\sin^4(x)\,dx =$

$$= \int \cos^2(x)\sin^4(x)\cdot \underline{\cos(x)\,dx}$$

$$= \int [1-\sin^2(x)]\cdot \sin^4(x)\cdot \underline{\cos(x)\,dx}$$

$$= \int [\sin^4(x) - \sin^6(x)]\cos(x)\,dx$$

Let $u = \sin(x)$

$du = \cos(x)\,dx$

$$= \int [u^4 - u^6]\,du$$

$$= \frac{u^5}{5} - \frac{u^7}{7} + C = \boxed{\frac{\sin^5(x)}{5} - \frac{\sin^7(x)}{7} + C}$$

(5) $\int \tan^3(x)\sec(x)\,dx =$

$$= \int \tan^2(x)\cdot \underline{\sec(x)\tan(x)\,dx}$$

$$= \int [\sec^2(x) - 1]\cdot \underline{\sec(x)\tan(x)\,dx}$$

Let $u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

$$= \int [u^2 - 1]\,du$$

$$= \frac{u^3}{3} - u + C = \boxed{\frac{\sec^3(x)}{3} - \sec(x) + C}$$

pJ Shree

⑥ $\displaystyle\int \frac{x}{2\sqrt{4-x^2}}\,dx$

NOTE: since the degree in the numerator is one less than the degree of the numerator we may simply use a u-subs.

Let $u = 4-x^2$
$du = -2x\,dx$
$-\frac{1}{2}du = x\,dx$

so $= \left(-\frac{1}{2}\right)\frac{1}{2}\displaystyle\int \frac{1}{\sqrt{u}}\,du$

$= -\frac{1}{4}\displaystyle\int u^{-1/2}\,du$

$= -\frac{1}{4}\,\frac{u^{1/2}}{1/2} + C$

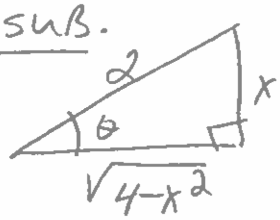
$= \left(-\frac{1}{4}\right)\left(\frac{2}{1}\right)\cdot\sqrt{4-x^2} + C$

$= \boxed{-\frac{1}{2}\sqrt{4-x^2} + C}$

#6 OR you could do this

$\displaystyle\int \frac{x}{2\sqrt{4-x^2}}\,dx$

TRIG. SUB.



Choose
$\sin(\theta) = \frac{x}{2}$
$\Rightarrow 2\sin(\theta) = x$
$\Rightarrow 2\cos(\theta)\,d\theta = dx$

Choose
$\cos(\theta) = \frac{\sqrt{4-x^2}}{2}$
$\Rightarrow 2\cos(\theta) = \sqrt{4-x^2}$

substituting:

$= \displaystyle\int \frac{2\sin(\theta)\,2\cos(\theta)}{2\cdot 2\cos(\theta)}\,d\theta$

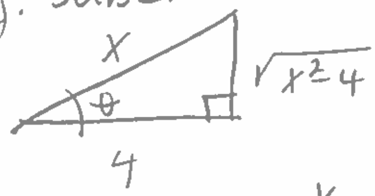
$= \displaystyle\int \sin(\theta)\,d\theta$

$= -\cos(\theta) + C$

$= \boxed{-\frac{\sqrt{4-x^2}}{2} + C}$

#7 $\displaystyle\int \frac{x^3}{\sqrt{x^2-4}}\,dx$

Trig. subs.



Choose: $\sec(\theta) = \frac{x}{4}$

$\Rightarrow 4\sec(\theta) = x$

$\Rightarrow 4\sec(\theta)\tan(\theta)\,d\theta = dx$

Choose $\tan(\theta) = \frac{\sqrt{x^2-4}}{4}$

$\Rightarrow 4\tan(\theta) = \sqrt{x^2-4}$

substituting

$\Longrightarrow \; = \displaystyle\int \frac{(4\sec(\theta))^3\, 4\sec(\theta)\tan(\theta)}{4\tan(\theta)}\,d\theta$

$= \displaystyle\int 64\sec^4(\theta)\,d\theta$

$= \displaystyle\int 64\sec^2(\theta)\,\underline{\sec^2(\theta)\,d\theta}$

$= 64\displaystyle\int [\tan^2(\theta)+1]\,\underline{\sec^2(\theta)\,d\theta}$

Let $u = \tan(\theta)$

$du = \sec^2(\theta)\,d\theta$

$= 64\displaystyle\int [u^2+1]\,du$

$= 64\frac{u^3}{3} + u + C$

$= \frac{64\tan^3(\theta)}{3} + \tan(\theta) + C$

$= \boxed{\frac{64}{3}\left(\frac{\sqrt{x^2-4}}{4}\right)^3 + \frac{\sqrt{x^2-4}}{4} + C}$

#8. $\int \frac{x-17}{x^2+x-6}\,dx = \int \frac{(x-17)}{(x+3)(x-2)}\,dx$

Par Frac

$\Rightarrow \frac{x-17}{(x+3)(x-2)} = \frac{A}{(x+3)} + \frac{B}{(x-2)}$

$\Rightarrow x-17 = A(x-2) + B(x+3)$

Let $x=2$

$\Rightarrow -15 = A(0) + B(5)$

$\Rightarrow B = -3$

Let $x=-3$

$\Rightarrow -20 = A(-5) + B(0)$

$\Rightarrow A = 4$

So we have

$= \int \left[\frac{4}{x+3} - \frac{3}{x-2}\right] dx = \boxed{4\ln|x+3| - 3\ln|x-2| + C}$

NOTE: THIS IS #10 NOT #9

$\int 2x\ln(x)\,dx$

Parts $u=\ln(x)$ $\quad v=x^2$

$du = \frac{1}{x}\,dx$ $\quad dv = 2x\,dx$

$= uv - \int v\,du$

$= x^2\ln(x) - \int x^2\cdot\left(\frac{1}{x}\right)dx$

$= x^2\ln(x) - \int x\,dx$

$= \boxed{x^2\ln(x) - \frac{x^2}{2} + C}$

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THIS is #9

$$\int \frac{4x^2-2x+9}{x^3+3x}\,dx = \int \frac{4x^2-2x+9}{x(x^2+3)}\,dx$$

Par Frac (TYPE III)

$$\frac{4x^2-2x+9}{x(x^2+3)} = \frac{A}{x} + \frac{(Bx+C)}{(x^2+3)}$$

$$\Rightarrow 4x^2-2x+9 = A(x^2+3) + (Bx+C)(x)$$

$$= Ax^2+3A+Bx^2+Cx$$

$$= (A+B)x^2 + Cx + 3A$$

EQUATE COEFFICIENTS

$$\Rightarrow A+B = 4$$

$$C = -2 \Rightarrow C=-2$$

$$3A = 9 \Rightarrow A=3$$

$3+B=4 \Rightarrow B=1$

SO we have:

SPLIT THE NUMERATOR

$$= \int\left[\frac{3}{x} + \frac{x-2}{x^2+3}\right]dx = \int\left[\frac{3}{x} + \frac{x}{x^2+3} - \frac{2}{x^2+3}\right]dx$$

SO WE GET

$$= \boxed{\ln|x| + \frac{1}{2}\ln|x^2+3| - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C}$$

$$\int \frac{3}{x}\,dx = \ln|x| + C_1$$

$$\int \frac{x}{x^2+3}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$$

Let $u = x^2+3$, $du = 2x\,dx$, $\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\ln|u| + C_2$$

$$= \frac{1}{2}\ln|x^2+3| + C_2$$

$$\int \frac{2}{x^2+3}\,dx = 2\int \frac{1}{x^2+3}\,dx$$

"$\tan^{-1}$ form" $u = x$, $a = \sqrt{3}$, $du = dx$

$$= 2\int \frac{1}{u^2+a^2}\,du$$

$$= 2\cdot\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C_3$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C_3$$

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III.

(11) $\lim_{x\to 1^+} \dfrac{1-x+\ln(x)}{x^3-3x+2} =$ direct substitution gives $\frac{0}{0}$ so

$\stackrel{L'H}{=} \lim_{x\to 1^+} \dfrac{0-1+\frac{1}{x}}{3x^2-3x}$ again, direct subst. gives $\frac{0}{0}$ so.

$\stackrel{L'H}{=} \lim_{x\to 1^+} \dfrac{-x^{-2}}{6x-3}$

$= \dfrac{-(1)^{-2}}{6(1)-3} = \dfrac{-1}{6-3} = \boxed{-\frac{1}{3}}$

(12) $\lim_{x\to 0} \dfrac{x^2-x}{\tan(x)}$ direct subst. gives $\frac{0}{0}$ so

$\stackrel{L'H}{=} \lim_{x\to 0} \dfrac{2x-1}{\sec^2(x)} = \dfrac{2(0)-1}{\sec^2(0)}$

$= \dfrac{-1}{1}$

$= \boxed{-1}$

III.

PG 8.jpg

(13) $$\int_{-1}^{\infty} e^{-x}\,dx = \lim_{b\to\infty}\left[\int_{-1}^{b} e^{-x}\,dx\right]$$

$$= \lim_{b\to\infty}\left[-e^{-x}\right]_{-1}^{b}$$

$$= \lim_{b\to\infty}\left[(-e^{-(b)}) - (-e^{-(-1)})\right]$$

$$= \lim_{b\to\infty}\left[\frac{-1}{e^{b}} + e^{1}\right]$$

$$= \cancel{\frac{1}{e^{\infty}}}^{\,0} + e$$

$$= \boxed{e}$$

(14) $$\int_{1}^{2} \frac{1}{(x-2)^2}\,dx = \lim_{b\to 2^-}\left[\int_{1}^{b} \frac{1}{(x-2)^2}\,dx\right]$$

has a vertical asymptote at $x=2$ so this is an improper integral

Let $u = x-2$

$du = dx$

$$= \lim_{b\to 2^-}\int_{c}^{d} u^{-2}\,du$$

$$= \lim_{b\to 2^-}\left[\frac{u^{-1}}{-1}\right]_{c}^{d}$$

$$= \lim_{b\to 2^-}\left[\frac{1}{-(x-2)}\right]_{1}^{b}$$

$$= \lim_{b\to 2^-}\left[\left(\frac{1}{-(b-2)}\right) - \left(\frac{1}{-(1-2)}\right)\right]$$

$$= \lim_{b\to 2^-}\left[\frac{1}{2-b} + \frac{1}{-1}\right]$$

$$= \frac{1}{0^+} - 1$$

$$= \infty - 1$$

$$= \boxed{\infty}$$

OR simply

$\boxed{\text{DIVERGES}}$

or simply

$\boxed{\text{D.N.E.}}$

EITHER IS OK

Pg 9me

(15) 

$$C = 4\int_0^r \sqrt{1+[f'(x)]^2}\,dx$$

$x^2+y^2=r^2$

$\Rightarrow y = \sqrt{r^2-x^2} = (r^2-x^2)^{1/2}$

$y' = \frac{1}{2}(r^2-x^2)^{-1/2}(-2x)$

$= \dfrac{-x}{\sqrt{r^2-x^2}}$

$[y']^2 = \dfrac{x^2}{r^2-x^2}$

$$= 4\int_0^r \sqrt{1+\frac{x^2}{r^2-x^2}}\,dx$$

$$= 4\int_0^r \sqrt{\frac{r^2-x^2}{r^2-x^2}+\frac{x^2}{r^2-x^2}}\,dx$$

$$= 4\int_0^r \sqrt{\frac{r^2\cancel{-x^2}\cancel{+x^2}}{r^2-x^2}}\,dx$$

$$= 4\int_0^r \frac{r}{\sqrt{r^2-x^2}}\,dx$$

$$= 4r\int_0^r \frac{1}{\sqrt{r^2-x^2}}\,dx$$

"$\sin^{-1}$ form" $a=r$ $u=x$
$du=dx$

$$= 4r\int_c^d \frac{1}{\sqrt{a^2-u^2}}\,du$$

$$= 4r\left[\sin^{-1}\left(\frac{u}{a}\right)\right]_c^d$$

$$= 4r\sin^{-1}\left(\frac{x}{r}\right)\Big|_0^r$$

$$= 4r\left[\left(\sin^{-1}\left(\frac{r}{r}\right)\right)-\left(\sin^{-1}\left(\frac{0}{r}\right)\right)\right]$$

$$= 4r\sin^{-1}(1)$$

$$= 4r\left(\frac{\pi}{2}\right) = \boxed{2\pi r}$$

SEE NEXT PAGES!!!

TWO EXTRA PROBLEMS TO CONSIDER

(YOU'LL BE GLAD YOU DID).

(16) $\displaystyle\int \frac{2x^4+10x^2+4}{x(x^2+2)^2}\,dx$ (17) $\displaystyle\lim_{x\to 0}\left(\cos(x)\right)^{\frac{1}{x^2}}$

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(16) $\displaystyle\int \frac{2x^4+10x^2+4}{x(x^2+2)^2}\,dx$

Par Frac (Type III)

$$\frac{2x^4+10x^2+4}{x(x^2+2)^2} = \frac{A}{x} + \frac{(Bx+C)}{(x^2+2)} + \frac{Dx+E}{(x^2+2)^2}$$

$$\Rightarrow 2x^4+10x^2+4 = A(x^2+2)^2 + (Bx+C)(x)(x^2+2) + (Dx+E)(x)$$

$$= A(x^4+4x^2+4) + (Bx+C)(x^3+2x) + Dx^2 + Ex$$

$$= Ax^4 + 4Ax^2 + 4A + Bx^4 + 2Bx^2 + Cx^3 + 2Cx + Dx^2 + Ex$$

$$= (A+B)x^4 + Cx^3 + (4A+2B+D)x^2 + (2C+E)x + 4A$$

EQUATE COEFFICIENTS:

$\Rightarrow$

$$\begin{aligned} A+B &= 2 \\ C &= 0 \longrightarrow E=0 \\ 4A+2B+D &= 10 \\ 2C+E &= 0 \\ 4A &= 4 \Rightarrow A=1 \text{ so with EQ 1} \Rightarrow B=1 \end{aligned}$$

so $4(1)+2(1)+D=10 \Rightarrow D=4$

WE GET $A=1$ $B=1$ $C=0$ $D=4$ $E=0$ $\Rightarrow$

(1b) cont:

so we have:

$$= \int \left[\frac{1}{x} + \frac{x}{x^2+2} + \frac{4x}{(x^2+2)^2} \right] dx$$

(a) (b) (c)

we have

(a) $\int \frac{1}{x} dx = \ln|x| + C_1$

(b) $\int \frac{x}{x^2+2} dx$

Let $u = x^2+2$

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$= \frac{1}{2} \int \frac{1}{u} du$

$= \frac{1}{2} \ln|u| + C_2$

$= \frac{1}{2} \ln|x^2+2| + C_2$

(c) $\int \frac{4x}{(x^2+2)^2} dx$

Let $u = x^2+2$

$du = 2x\,dx$

$2\,du = 4x\,dx$

$= 2\int u^{-2} du = 2\frac{u^{-1}}{-1} + C_3$

$= -2(x^2+2)^{-1} + C_3$

so combining these we get

$$= \boxed{\ln|x| + \frac{1}{2}\ln|x^2+2| - \frac{2}{(x^2+2)} + C}$$

#17 $\lim_{x \to 0} (\cos(x))^{\frac{1}{x^2}} \rightarrow 1^{\infty}$

Let $y = \lim_{x \to 0} (\cos(x))^{\frac{1}{x^2}}$

$$\Rightarrow \ln(y) = \ln\left[\lim_{x \to 0} (\cos(x))^{\frac{1}{x^2}}\right]$$

$$= \lim_{x \to 0}\left[\ln(\cos(x))^{\frac{1}{x^2}}\right]$$

$$= \lim_{x \to 0}\left[\frac{1}{x^2} \cdot \ln(\cos(x))\right]$$

$$= \lim_{x \to 0}\left[\frac{\ln(\cos(x))}{x^2}\right] \longrightarrow$$ direct subst gives $\frac{0}{0}$

$$\stackrel{L'H}{=} \lim_{x \to 0}\left[\frac{\frac{1}{\cos(x)}(-\sin(x))}{2x}\right]$$

$$= \lim_{x \to 0}\left[\frac{-\tan(x)}{2x}\right] \longrightarrow$$ direct subst gives $\frac{0}{0}$

$$\stackrel{L'H}{=} \lim_{x \to 0}\left[\frac{-\sec^2(x)}{2}\right]$$

$$= \frac{-\sec^2(0)}{2}$$

$$= \boxed{\frac{-1}{2}}$$