

SOLUTIONS:

① $f(x) = \frac{2x-6}{x+3}$

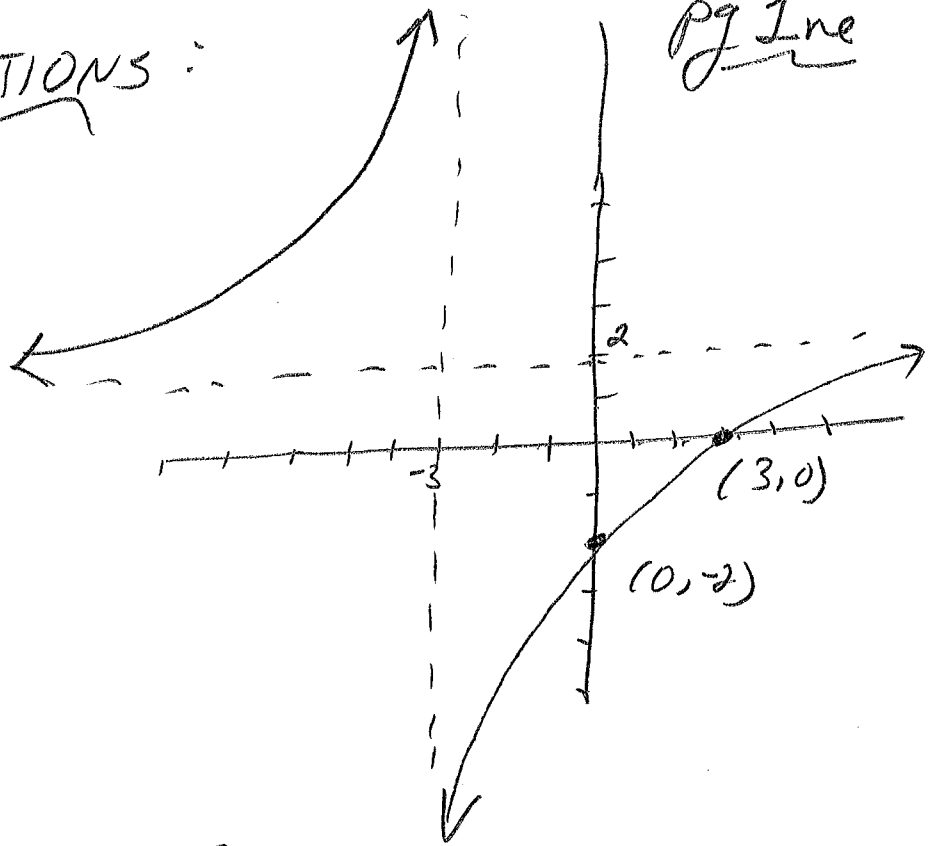
⊗ Vertical
Asymptotes
set denom = 0

$$\Rightarrow x+3=0$$

$$\Rightarrow x = -3$$

⊗ Horizontal
Asymptote.
Compare degrees

num $\rightarrow 1 \Rightarrow$ degrees equal
denom $\rightarrow 1$ form fraction of
coefficients $\Rightarrow y = 2$



⊗ y-int: set $x=0$

$$f(0) = \frac{2(0)-6}{(0)+3} = \frac{-6}{3} = -2 \Rightarrow (0, -2)$$

⊗ x-int: set numerator = 0

$$\Rightarrow 2x-6=0 \Rightarrow 2x=6 \Rightarrow x=3 \Rightarrow (3, 0)$$

⊗ on the left-hand side of the asymptote,
Note that there is no x-intercept $\Rightarrow$ Hence it must be "upper left"

⊗ on the right-hand side of the asymptote:
Note the graph must pass thru the intercepts: Hence it must be "lower right"

② $f(x) = \frac{x}{x-4}$

⊗ Vertical asymp.

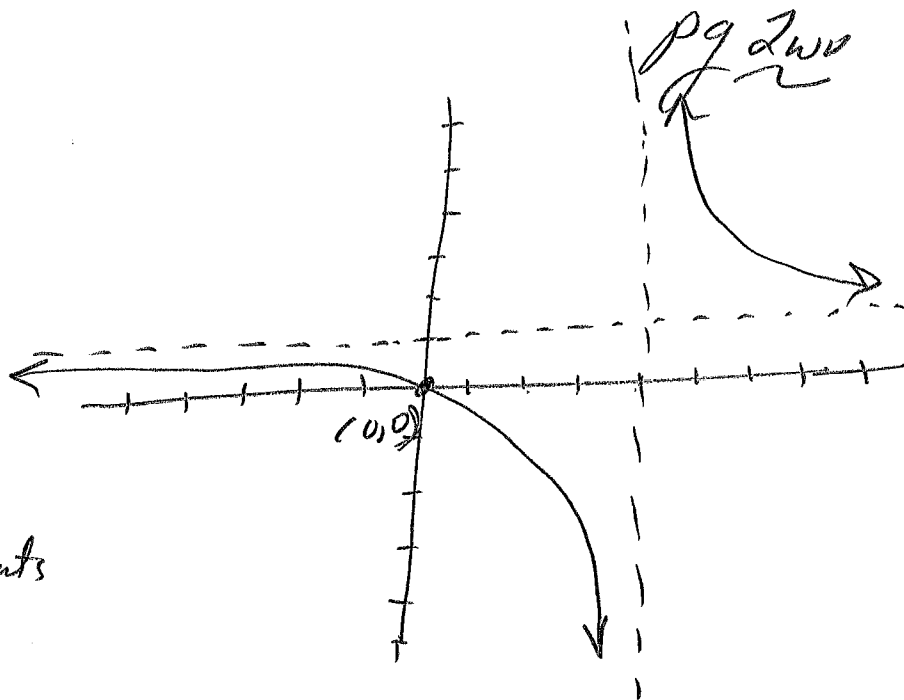
set denom = 0

$\Rightarrow x-4=0 \Rightarrow x=4$

⊗ Horiz. asymp.

compare degrees

$\Rightarrow \left. \begin{array}{l} \text{num} \rightarrow 1 \\ \text{denom} \rightarrow 1 \end{array} \right\} \Rightarrow \begin{array}{l} \text{Form fraction} \\ \text{of coefficients} \end{array}$
 $\Rightarrow \frac{1}{1} = 1$



⊗ y-int: set $x=0$

$\Rightarrow f(0) = \frac{0}{0-4} = \frac{0}{-4} = 0 \Rightarrow (0,0)$

⊗ x-int: set num = 0

$\Rightarrow x=0 \Rightarrow (0,0)$

⊗ on the left-hand side of the asymptote
the graph must pass thru $(0,0) \Rightarrow$ "lower left"

⊗ on the right-hand side of the asymptote
the graph must NOT cross the x-axis (No intersect)
so $\Rightarrow$ "upper right"

③ $f(x) = \frac{12}{x-4}$

⊗ Vertical asymp. set denom = 0

$\Rightarrow x-4=0 \Rightarrow x=4$

⊗ horizontal asymp: compare degrees

$\Rightarrow \left. \begin{array}{l} \text{num} \rightarrow 0 \\ \text{denom} \rightarrow 1 \end{array} \right\} \Rightarrow \begin{array}{l} \text{numerator} \\ \text{smaller} \end{array} \Rightarrow \underline{\underline{x\text{-axis}}}$

⊗ y-int: set $x=0$

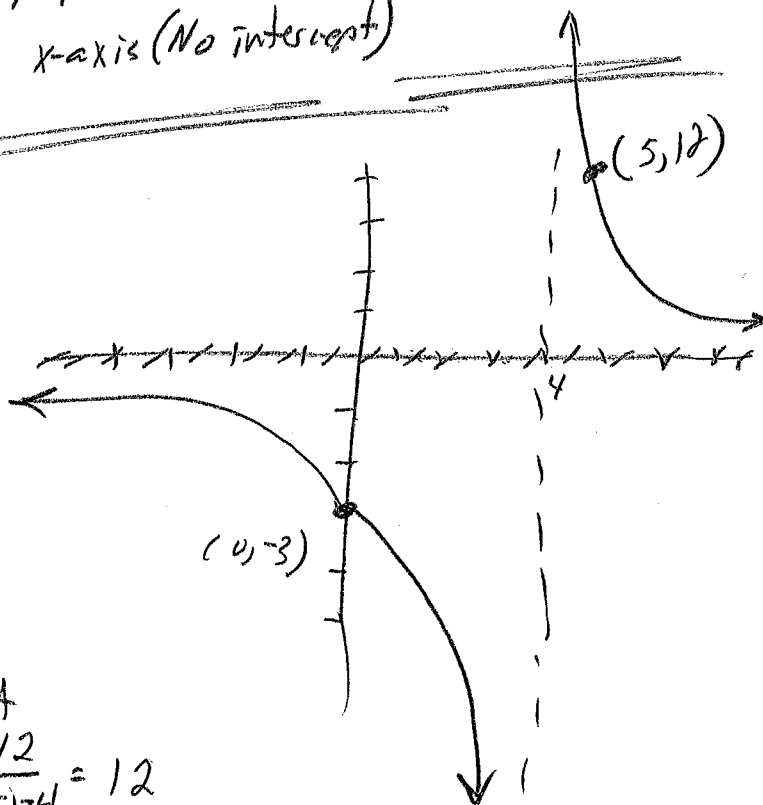
$\Rightarrow f(0) = \frac{12}{0-4} = \frac{12}{-4} = -3 \Rightarrow (0,-3)$

⊗ x-int: set num = 0

$\Rightarrow 12=0 \Rightarrow \text{No sol'n} \Rightarrow \text{No x-int.}$

Plot point

$f(5) = \frac{12}{(5)-4} = 12$



④ $f(x) = \frac{x^2 - 1}{x^2 - 9}$

⊛ Vert. asympt.: set denom = 0

$$\Rightarrow x^2 - 9 = 0$$

$$\Rightarrow (x+3)(x-3) = 0$$

$$\Rightarrow x = -3 \quad x = 3 \Rightarrow$$

⊛ Horiz. Asymp.: ~~set~~

Compare degrees

$$\Rightarrow \text{num} \rightarrow 2 \Rightarrow \text{degrees}$$

$$\text{denom} \rightarrow 2 \Rightarrow \text{equal}$$

$\Rightarrow$ fraction of coefficients:

$$\Rightarrow \frac{1}{1} \Rightarrow \text{at } 1$$

⊛ y-int set $x=0$

$$\Rightarrow f(0) = \frac{(0)^2 - 1}{(0)^2 - 9} = \frac{-1}{-9} = \frac{1}{9}$$

$$(0, \frac{1}{9})$$

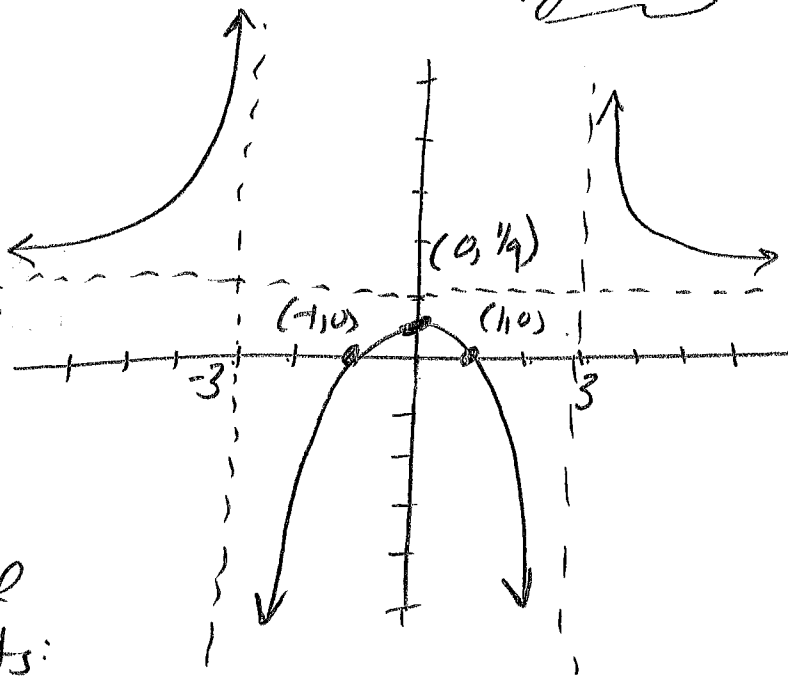
⊛ x-int: set num = 0

$$\Rightarrow x^2 - 1 = 0 \Rightarrow$$

$$\Rightarrow x = \pm 1$$

$$(1, 0) \quad (-1, 0)$$

⊛ graph



⑤. $f(x) = \frac{x-2}{x^2-16}$

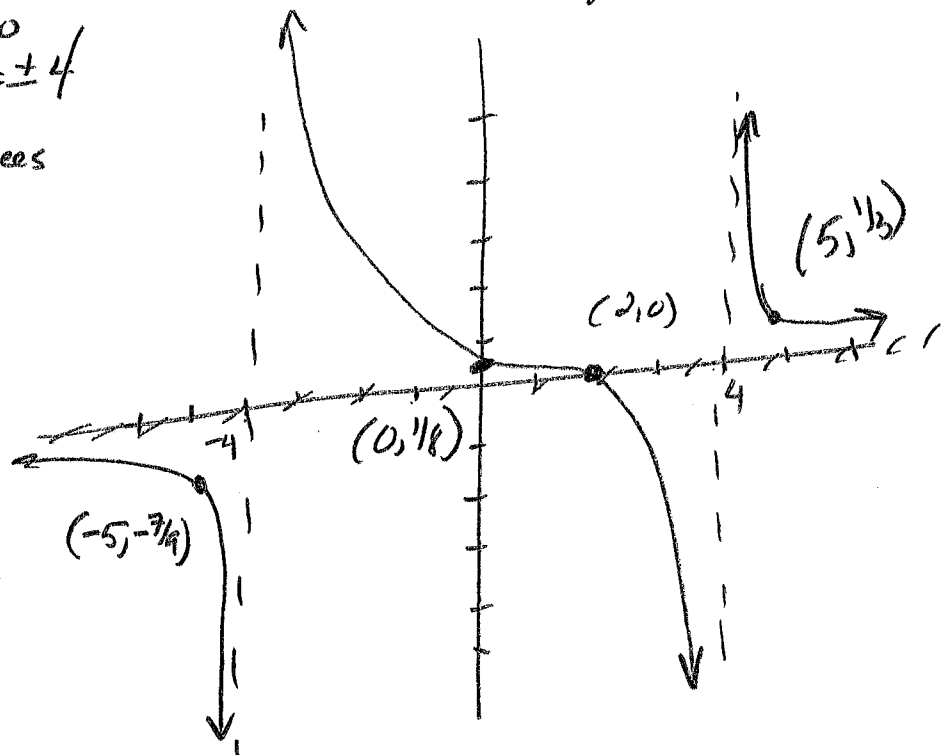
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⑥ Vert asymp. set denom = 0
 $\Rightarrow x^2-16=0 \Rightarrow x=\pm 4$

⑦ Horiz. asymp. compare degrees
 $\Rightarrow \text{num} \rightarrow 1$
 $\Rightarrow \text{denom} \rightarrow 2 \rightarrow \text{numerator is smaller}$
 $\Rightarrow x\text{-axis}$

⑧ y-int: set $x=0$
 $\Rightarrow f(0) = \frac{(0)-2}{(0)^2-16} = \frac{-2}{-16} = \frac{1}{8}$
 $(0, 1/8)$

⑨ x-int: set num = 0
 $x-2=0 \Rightarrow x=2$
 $(2, 0)$



FAR LEFT: $f(-5) = \frac{(-5)-2}{(-5)^2-16} = \frac{-7}{9} \Rightarrow (-5, -7/9)$

FAR RIGHT: $f(5) = \frac{(5)-2}{(5)^2-16} = \frac{3}{9} = \frac{1}{3} \Rightarrow (5, 1/3)$

$$\textcircled{6} f(x) = \frac{x^2 - 2x - 3}{x^2 + x - 6}$$

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* Vert. asymp. $\Rightarrow$ set denom $= 0$

$$\Rightarrow x^2 + x - 6 = 0$$

$$\Rightarrow (x+3)(x-2) = 0$$

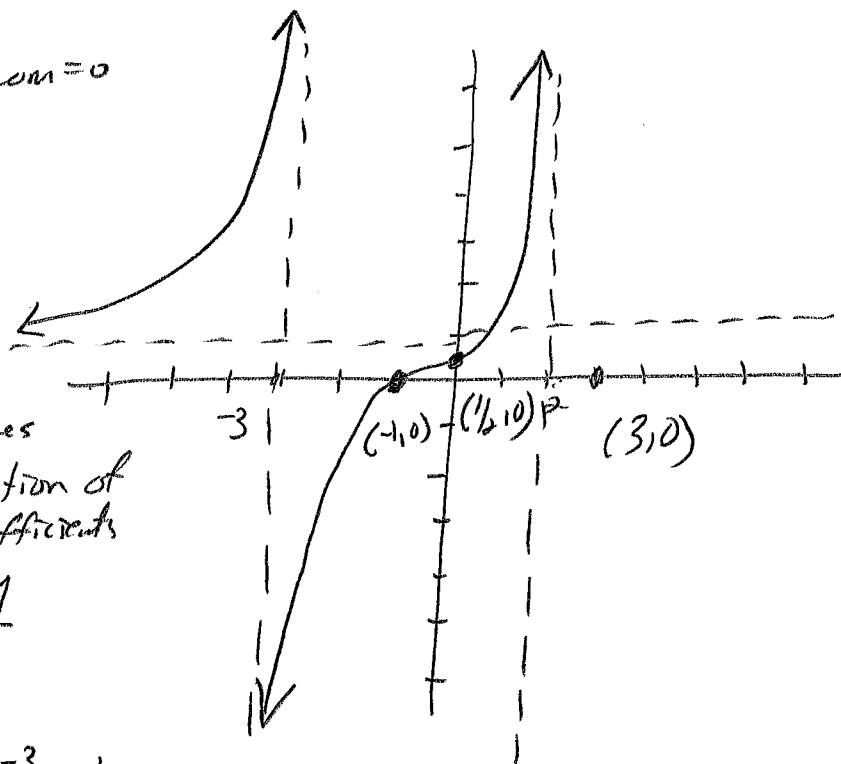
$$\Rightarrow x = -3 \text{ or } x = 2$$

* horiz. asymp. ~~$\Rightarrow$ set num $= 0$~~

$\Rightarrow$ compare degrees

num $\rightarrow 2$ } degrees
denom $\rightarrow 2$ } equal $\Rightarrow$ fraction of coefficients

$$\Rightarrow \frac{1}{1} = 1$$



* y-int: set $x = 0$

$$\Rightarrow f(0) = \frac{(0)^2 - 2(0) - 3}{(0)^2 + (0) - 6} = \frac{-3}{-6} = \frac{1}{2}$$

$\Rightarrow (0, \frac{1}{2})$

* x-int: set num $= 0$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x-3)(x+1) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -1 \Rightarrow (3, 0) \text{ } (-1, 0)$$

* Note on the far left there is No x-intercept.