(I) **SPRING #1:** Find k

$16 = \int_0^4 kx\,dx \Rightarrow 16 = \left[\frac{kx^2}{2}\right]_0^4$

$\Rightarrow 16 = \left[\left(\frac{k(16)}{2}\right) - \left(\frac{k(0)^2}{2}\right)\right]$

$\Rightarrow 16 = 8k$

$\Rightarrow k = 2$ so $F(x) = 2x$

SPRING #2: Find k

$F = kx \Rightarrow 36 = k(6) \Rightarrow k = 6 \Rightarrow F(x) = 6x$

so (A) $W = \int_2^5 2x\,dx = \left[\frac{2x^2}{2}\right]_2^5 = \left[(5)^2 - (2)^2\right] = 25 - 4 = 21$

(B) $W = \int_1^3 6k\,dx = \left[\frac{6k^2}{2}\right]_1^3 = \left[(3(3)^2) - (3(1)^2)\right] = 27 - 3 = 24$

so $\boxed{\boxed{B} \text{ requires more work}}$

(II). $\Delta F = (\text{density})(\text{depth})\underbrace{(\text{Length})}_{(\text{right}-\text{left})}(\text{width})$

Note: we need the equations in terms of y:

Left: $y = 2\sqrt{x+1} - 4$

$\Rightarrow y + 4 = 2\sqrt{x+1}$

$\Rightarrow \frac{y+4}{2} = \sqrt{x+1}$

$\Rightarrow \left(\frac{y+4}{2}\right)^2 = x+1 \Rightarrow x = \left(\frac{y+4}{2}\right)^2 - 1$

right: $y = -x - 2$

$\Rightarrow y + 2 = -x$

$\Rightarrow -y - 2 = x$

so $\boxed{F = \rho \int_{-4}^{-2} (-y)\left[(-y-2) - \left(\left(\frac{y+4}{2}\right)^2 + 1\right)\right] dy}$

(III). $y = 1 + \sin(2x)$

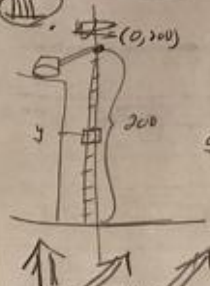
$y' = \cos(2x)(2)$

$= 2\cos(2x)$

so $\boxed{S = \int_0^{\pi} \sqrt{1 + (2\cos(2x))^2}\,dx}$

This is the Arc Length letter

(IIII).



$\Delta W = (\Delta F)(\text{Distance lifted})$

$= \underbrace{(\text{weight})(\text{length})}_{}(\text{distance lifted})$

given $= (6)(\Delta y)$ $\quad (200 - y)$

$\Rightarrow W = \boxed{\int_0^{200} 6(200-y)\,dy}$

NOTE!! we did Not discuss this type of problem this semester, so I will not ask you one like it.

(V). (A) While either shells or disk/washers may be used, the disk/washer method would require partitioning the y-axis and hence introduces the cusp at (0,1). It is thus much easier to use the shell method, so partition the x-ax from 0 to $\pi/2$

cont.

V cont. SHELLS PARTITION THE X-AXIS. pg 3 three

(A)

$\Delta V = (2\pi)(\text{height})(\text{radius})(\text{thickness})$

$= (2\pi)(\text{top} - \text{bottom})(\text{radius})(\text{thickness})$

$= (2\pi)\big((1+\sin(x)) - (1-\cos(2x))\big)\left(\tfrac{\pi}{2} - x\right)(\Delta x)$

so

$$\boxed{V = 2\pi \int_0^{\pi/2} \left[(1+\sin(x)) - (1-\cos(2x))\right]\left(\tfrac{\pi}{2} - x\right) dx}$$

(B). IN THIS CASE USING THE SHELLS REQUIRES Partitioning the y-axis which creates a cusp at (0,1). Hence I choose Disk/washer method.

Partition x-axis from 0 to $\pi/2$

$\Delta V = \pi\left(\left(\text{OUTER RADIUS}\right)^2 - \left(\text{INNER RADIUS}\right)^2\right)(\text{thickness})$

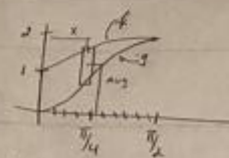
Since the figure is Hollow

so

$$\boxed{V = \pi \int_0^{\pi/2} \left[(1+\sin(x))^2 - (1-\cos(2x))^2\right] dx}$$

VI pg 4 four

Let density = δ



$\Delta M_x = (\text{density})(\text{distance to x-axis})(\text{Length})(\text{thickness})$

$= (\delta)\left(\text{average of function values}\right)\left(\text{top minus bottom}\right)(\Delta x)$

so

$$\boxed{M_x = \delta \int_0^{\pi/2} \frac{\left[(1+\sin(x)) + (1-\cos(2x))\right]}{2}\left[(1+\sin(x)) - (1-\cos(2x))\right] dx}$$

$\Delta M_y = (\text{density})(\text{distance to y-axis})(\text{length})(\text{thickness})$

$= (\text{density})(x)\big((\text{top}) - (\text{bottom})\big)\,\Delta x$

$$= \boxed{\delta \int_0^{\pi/2} (x)\left[(1+\sin(x)) - (1-\cos(2x))\right] dx}$$

so

$$\bar{x} = \frac{M_y}{m} = \frac{\delta \int_0^{\pi/2} (x)\left[(1+\sin(x)) - (1-\cos(2x))\right] dx}{\delta \int_0^{\pi/2} \left[(1+\sin(x)) - (1-\cos(2x))\right] dx}$$

$$\bar{y} = \frac{M_x}{m} = \frac{\delta \int_0^{\pi/2} \frac{\left[(1+\sin(x)) + (1-\cos(2x))\right]}{2}\left[(1+\sin(x)) - (1-\cos(2x))\right] dx}{\delta \int_0^{\pi/2} \left[(1+\sin(x)) - (1-\cos(2x))\right] dx}$$

pg 5ive

(VII) $f(x) = 1 + x^{2/3}$ on $[-1, 8]$

$f'(x) = \frac{2}{3}x^{-1/3}$

So $\boxed{S = \int_{-1}^{0}\sqrt{1+\left(\frac{2}{3}x^{-1/3}\right)^2}\,dx + \int_{0}^{8}\sqrt{1+\left(\frac{2}{3}x^{-1/3}\right)^2}\,dx}$

This is the Arc Length letter

VIII (A) ⓐ IF DISK METHOD PARTITION y-axis from 0 to 1

must have $y = \tan(x)$

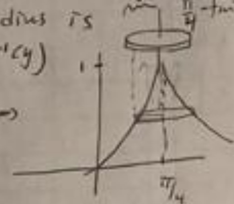
$\Rightarrow \tan^{-1}(y) = x$

So $V = \pi\int_0^1 (\text{radius})^2\,dy$

$= \boxed{\pi\int_0^1 \left(\frac{\pi}{4} - \tan^{-1}(y)\right)^2 dy}$

note: the radius is $1 - \tan^{-1}(y)$

↳ see pic →



or ⓑ IF SHELL METHOD

↳ Partition the x-axis from 0 to $\pi/4$

So $V = 2\pi\int_0^{\pi/4} (\text{radius})(\text{height})\,dx$

$= \boxed{2\pi\int_0^{\pi/4}\left(\frac{\pi}{4} - \tan(x)\right)\left(\tan(x)\right)dx}$

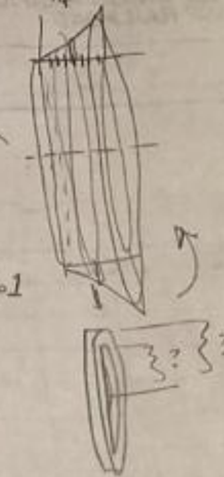
VIII (B) ⓐ washer/disk method.

pg 6ix

$\Rightarrow$ partition x-axis from 0 to $\pi/4$

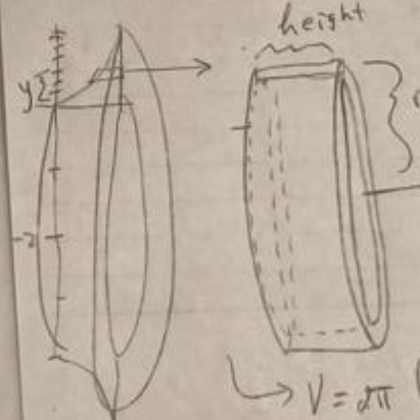
$V = \pi\int_0^{\pi/4}\left[\left(\text{outer radius}\right)^2 - \left(\text{inner radius}\right)^2\right]dx$

$= \pi\int_0^{\pi/4}\left[(\tan(x)+2)^2 - (0+2)^2\right]dx$



or ⓑ shell method

$\Rightarrow$ partition the y-axis from 0 to 1

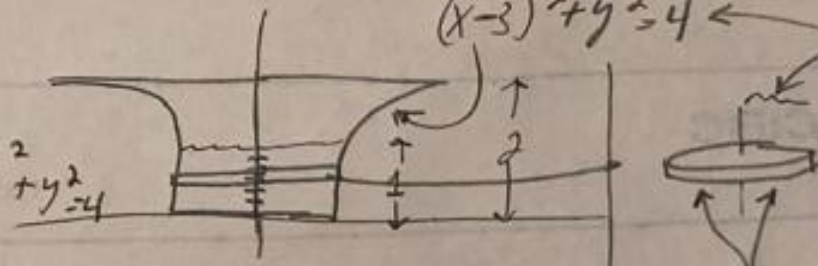


↳ $V = 2\pi\int_0^1 (\text{radius})\underbrace{(\text{height})}_{\text{Right - left}}\,dy$

$= \boxed{2\pi\int_0^1 (y+2)\left(\left(\frac{\pi}{4} - \tan^{-1}(y)\right)\right)dy}$

IX. $W =$

$(x+3)^2+y^2=4$ $\qquad$ $(x-3)^2+y^2=4$ $\qquad$ pg 7even



radius

NOTE:
Because of symmetry these two radii are equal

$$W = (\text{density})(\text{distance})\underbrace{(\text{Volume})}_{\underbrace{(\text{area})}_{(\pi)(\text{radius})^2}(\text{thickness})}$$

$$\Rightarrow \Delta W = (\rho)(2-y)(\pi)\left(\sqrt{4-y^2}-3\right)^2(\Delta y)$$

NOTE:

$$(x-3)^2+y^2=4$$
$$\Rightarrow (x+3)^2 = 4-y^2$$
$$\Rightarrow x+3 = \sqrt{4-y^2}$$
$$\Rightarrow x = \sqrt{4-y^2}-3$$

so

$$\boxed{W = \rho\pi\int_0^1 (2-y)\left(\sqrt{4-y^2}-3\right)^2 dy}$$