I

Spring #1

$$64 = \int_0^4 kx\,dx$$

$$\Rightarrow 64 = \left[\frac{kx^2}{2}\right]_0^4$$

$$\Rightarrow 64 = \left[\left(\frac{k(16)}{2}\right) - \left(\frac{k(0)}{2}\right)\right]$$

$$\Rightarrow 64 = 8k$$

$$\Rightarrow k = 8$$

so $F_1(x) = 8x$

Spring #2

$$18 = k(5-2)$$

$$= k(3)$$

$$k = 6$$

$$F_2(x) = 6x$$

A $W = \int_2^3 8x\,dx$

$$= \left[\frac{8x^2}{2}\right]_2^3$$

$$= (4(3)^2) - (4(2)^2)$$

$$= 36 - 16$$

$$= 20$$

B $W = \int_1^4 6x\,dx$

$$= \left[\frac{6x^2}{2}\right]_1^4$$

$$= (3(4)^2) - (3(1)^2)$$

$$= 48 - 3$$

$$= 45$$

$\boxed{B}$ REQUIRES MORE WORK

$16 - y^2 + 8y - 16$

$(-y^2 + 8y)$

(II) $\Delta W = (\delta)(4-y)(\pi)\left(\sqrt{16-(y-4)^2}\right)^2 \Delta y$

$\Rightarrow W = \boxed{\delta\pi \int_0^1 (4-y)\left(\sqrt{16-(4-y)^2}\right)^2 dy}$

$y = \sqrt{x} - 3 \qquad y = -2x - 3$

$y + 3 = \sqrt{x} \qquad y + 3 = -2x$

$(y+3)^2 = x \qquad \frac{y+3}{-2} = x$

(III) $\Delta F =$ (density)(depth)(area) (length)(width) (RT − LFT) (Δy)

$= \delta(-y)\left((y+3)^2 - \left(\frac{y+3}{2}\right)\right)\Delta y$

so $\boxed{F = \delta \int_{-3}^{-1} (-y)\left((y+3)^2 - \left(\frac{y+3}{-2}\right)\right) dy}$

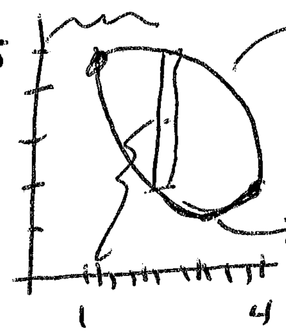
(IIII) $y = (x^2-9)^{2/3} \Rightarrow y' = \underline{2/3\,(x^2-9)^{-1/3}(2x)}$

so

$$S = \int_{-1}^{3} \sqrt{1 + \left[2/3\,(x^2-9)^{-1/3}(2x)\right]^2}\, dx + \int_{3}^{4} \sqrt{1 + \left[2/3\,(x^2-9)^{-1/3}(2x)\right]^2}\, dx$$

The Arc Length letter

IV



TOP $\rightarrow (x-1)^2+(y-2)^2=9 \Rightarrow (y-2)^2=9-(x-1)^2$

$\Rightarrow y-2=\sqrt{9-(x-1)^2}$

$\Rightarrow y=2+\sqrt{9-(x-1)^2}$

BOTTOM: $y=(x-3)^2+1$

So

$$M_x=\delta\int_1^4 \frac{\left[\left(2+\sqrt{9-(x-1)^2}\right)+\left((x-3)^2+1\right)\right]}{2}\left[\left(2+\sqrt{9-(x-1)^2}\right)-\left((x-3)^2+1\right)\right]dx$$

$$M_y=\delta\int_1^4 (x)\left[\left(2+\sqrt{9-(x-1)^2}\right)-\left((x-3)^2+1\right)\right]dx$$

$$\bar{x}=\frac{\delta\int_1^4 (x)\left[\left(2+\sqrt{9-(x-1)^2}\right)-\left((x-3)^2+1\right)\right]dx}{\delta\int_1^4\left[\left(2+\sqrt{9-(x-1)^2}\right)-\left((x-3)^2+1\right)\right]dx}$$

$$\bar{y}=\frac{\delta\int_1^4 \frac{\left[\left(2+\sqrt{9-(x-1)^2}\right)+\left((x-3)^2+1\right)\right]}{2}\left[\left(2-\sqrt{9-(x-1)^2}\right)-\left((x-3)^2+1\right)\right]dx}{\delta\int_1^4\left[\left(2+\sqrt{9-(x-1)^2}\right)-\left((x-3)^2+1\right)\right]dx}$$

VI $y = x + \cos(x)$ on $[-\pi, 3\pi/2]$

$y' = 1 - \sin(x)$

So

$$\boxed{S = \int_{-\pi}^{3\pi/2} \sqrt{1 + [1-\sin(x)]^2}\, dx}$$

$\uparrow$ The arc length letter

VII A Shells

$$\boxed{V = 2\pi \int_0^{\pi/4} (x)\left[(1+\sin(2x)) - (2\tan(x))\right] dx}$$

$y = 1+\sin(2x)$ $\quad y = 2\tan(x)$

$y - 1 = \sin(2x)$ $\quad \frac{1}{2}y = \tan(x)$

$\sin^{-1}(y-1) = 2x$ $\quad \tan^{-1}(\frac{1}{2}y) = x$

$\frac{1}{2}\sin^{-1}(y-1) = x$

Disk/Washers

$$\boxed{V = \pi \int_0^1 (\tan^{-1}(\tfrac{1}{2}y))^2 dy + \pi \int_1^2 \left[(\tan^{-1}(\tfrac{1}{2}y))^2 - (\tfrac{1}{2}\sin^{-1}(y-1))^2\right] dy}$$

B. Shells

$$\boxed{V = 2\pi \int_0^1 \tan^{-1}(\tfrac{1}{2}y)(y+1)\, dy + 2\pi \int_0^1 \left[(\tan^{-1}(\tfrac{1}{2}y)) - (\tfrac{1}{2}\sin^{-1}(y-1))\right](y+1)\, dy}$$

Disk/Washer

$$\boxed{V = \pi \int_0^{\pi/4} \left[(1+\sin(2x)+1)^2 - (2\tan(x)+1)^2\right] dx}$$

VIII disk/washer method

$$V = \pi\int_0^2 \left[\left(\sqrt{\frac{y-2}{-2}}+3\right)^2 - \left(\sqrt{\tfrac{1}{2}y}+2\right)^2\right] dy \,+$$

$$+\ \pi\int_2^4 \left[\left(\frac{y-8}{-2}\right)^2 - \left(\tfrac{1}{2}y\right)^2\right] dy$$

OR

SHELLS

$$V = 2\pi\int_1^2 (x)\left[(2x) - (2(x-2)^2)\right] dx + 2\pi\int_2^3 (x)\left[(-2x+8) - (-2(x-3)^2+2)\right] dx$$

NOTE: COMPUTATIONS FOR THE DISK/WASHER METHOD:

⊛ $y = 2x$

$\Rightarrow \tfrac{1}{2}y = x$ ✓

⊛ $y = -2x+8$

$\Rightarrow y-8 = -2x$

$\Rightarrow \dfrac{y-8}{-2} = x$ ✓

⊛ $y = 2(x-2)^2$

$\Rightarrow \tfrac{1}{2}y = (x-2)^2$

$\Rightarrow \sqrt{\tfrac{1}{2}y} = x-2$

$\Rightarrow \sqrt{\tfrac{1}{2}y}+2 = x$ ✓

⊛ $y = -2(x-3)^2+2$

$\Rightarrow y-2 = -2(x-3)^2$

$\Rightarrow \dfrac{y-2}{-2} = (x-3)^2$

$\Rightarrow \sqrt{\dfrac{y-2}{-2}} = x-3$

$\Rightarrow \sqrt{\dfrac{y-2}{-2}}+3 = x$ ✓