

CALC I EXAM I SOLUTIONS FORM (A)

Pg 1 ne

① C

② C $\lim_{x \rightarrow \pi} \tan(x) = \tan(\pi) = 0$ so NOT B

$$\lim_{x \rightarrow 1} \frac{(x-2)}{(x^2-4)} = \frac{(1)-2}{(1)^2-4} = \frac{-1}{-3} = \frac{1}{3} \text{ so NOT D}$$

$$\lim_{x \rightarrow (-3)} \frac{(x^2-9)}{(x+3)} \Rightarrow \frac{(-3)^2-9}{(-3)+3} \rightarrow \frac{0}{0} \rightarrow \text{REQUIRES MORE WORK}$$

③ C $\lim_{x \rightarrow 4^+} f(x) \stackrel{\text{BOT}}{=} \lim_{x \rightarrow 4^+} (2x-5) = 2(4)-5 = 3$

④ C

search for δ : $|f(x) - L| < \epsilon \Rightarrow |(-6x-1) - (-13)| < \epsilon \Rightarrow$
 $\Rightarrow |-6x-1+13| < \epsilon \Rightarrow |-6x+12| < \epsilon \Rightarrow |-6||x-2| < \epsilon \Rightarrow$
 $\Rightarrow |x-2| < \frac{\epsilon}{|-6|} \text{ choose } \delta = \frac{\epsilon}{|-6|}$

⑤ E

$2x-7$ is a polynomial so no discontinuities if $x < 5$

$\frac{-8}{x-7} \Rightarrow x-7=0 \Rightarrow x=7$ one discontinuity so far.

must check $x=5$ $\textcircled{a} f(5) \stackrel{\text{TOP}}{=} 2(5)-7 = 3$ — $\textcircled{b} \lim_{x \rightarrow 5^-} f(x)$

$\textcircled{a} \lim_{x \rightarrow 5^-} f(x) \stackrel{\text{TOP}}{=} \lim_{x \rightarrow 5^-} (2x-7) = 2(5)-7 = 3$ —

$\textcircled{b} \lim_{x \rightarrow 5^+} f(x) \stackrel{\text{BOT}}{=} \lim_{x \rightarrow 5^+} \frac{-8}{x-7} = \frac{-8}{(5)-7} = \frac{-8}{-2} = 4$ —

so $\lim_{x \rightarrow 5} f(x)$ D.N.E $\Rightarrow$ discontinuity at $x=5$

so there are 2 total discontinuities

⑥ D

The unrestricted $\lim_{x \rightarrow 7} f(x) = 8$ forces both one-sided limits to agree and equal 8.

⑦ B For some reason I included the same question twice. pg 2nd

⑧ B $0 < |x-a| < \delta \Rightarrow 0 < |x-a| \Rightarrow x \neq a$.

⑨ E $\lim_{x \rightarrow \pi/2^-} f(x) \stackrel{\text{Top}}{=} \lim_{x \rightarrow \pi/2^-} (1 + 2x - \tan(2x)) = 1 + 2(\frac{\pi}{2}) - \tan(2(\frac{\pi}{2})) = 1 + \pi + \tan(\pi) = \underline{1 + \pi}$

$\lim_{x \rightarrow \pi/2^+} f(x) \stackrel{\text{Bot}}{=} \lim_{x \rightarrow \pi/2^+} (2x - \cos(2x)) = 2(\frac{\pi}{2}) - \cos(2(\frac{\pi}{2})) = \pi - \cos(\pi) = \pi - (-1) = \underline{1 + \pi}$

⑩ E the value of f at $x=7$ has no influence on the limit values.

⑪ B $\lim_{x \rightarrow 10} \frac{(\sqrt{x+15} - 6)}{(x-9)} = \frac{\sqrt{10+15} - 6}{(10-9)} = \frac{\sqrt{25} - 6}{1} = \frac{-1}{1} = -1$

⑫ E NOT $x=6$ because that is not in the region $x \leq 4$

yes at $x=5$. $\lim_{x \rightarrow 4^-} (2 - \frac{16}{(4-6)^2}) = 2 - \frac{16}{4} = -2$

$\lim_{x \rightarrow 4^+} (2(4) + \frac{16}{(4)-5}) = 8 + (-16) = -8$ } NOT at $x=4$

so the only discontinuity is at $x=5$

⑬ C Discontinuous if $\cos(x) = 0 \Rightarrow$ odd multiples of $\frac{\pi}{2}$
 $\Rightarrow (2k+1)\frac{\pi}{2}$

⑭ A continuity requires all these to exist at $x=-3$

(15) C We discussed this exact problem in class.

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$$\text{II } (16) \lim_{x \rightarrow a} [f(x) \pm g(x)] = \left[\lim_{x \rightarrow a} f(x) \right] \pm \left[\lim_{x \rightarrow a} g(x) \right]$$

$$(17) \lim_{x \rightarrow a} k = k$$

$$(18) \lim_{x \rightarrow a} [k f(x)] = k \left[\lim_{x \rightarrow a} f(x) \right]$$

(19) ① $f(a)$ is defined

② $\lim_{x \rightarrow a} f(x)$ is defined

$$(3) \lim_{x \rightarrow a} f(x) = f(a)$$

$$\text{III } (16) x^2 - 4 = 0$$

$$(7pts) \Rightarrow (x+2)(x-2) = 0$$

$$\Rightarrow \underline{x=2} \quad \cancel{x=-2}$$

not in the restricted interval

$$x \leq 1$$

Must check $x=1$

$$(1) f(1) \stackrel{\text{TOP}}{=} \frac{-3}{(1)^2 - 4} = \frac{-3}{-3} = 1 \checkmark$$

$$(ii) \lim_{x \rightarrow 1} f(x) \quad \text{a} \quad \lim_{x \rightarrow 1^-} \left(\frac{-3}{(1)^2 - 4} \right) = 1 \checkmark$$

$3 - x^2$ is a polynomial
 $\Rightarrow$ none here

$$(6) \lim_{x \rightarrow 1^+} (3 - (x)^2) = 3 - 1 = 2 \checkmark$$

so $\lim_{x \rightarrow 1} f(x)$ D.N.E.

Discontinuity at $x=1$

so NOT continuous at $x=1, x=-2$

(17) Continuous if $\lim_{x \rightarrow (-2)^-} f(x) = \lim_{x \rightarrow (-2)^+} f(x)$

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(7 pts)

$$\Rightarrow \lim_{x \rightarrow (-2)^-} (x^2 - kx) = \lim_{x \rightarrow (-2)^+} (4x + 2)$$

$$\Rightarrow (-2)^2 - k(-2) = 4(-2) + 2$$

$$\Rightarrow 4 + 2k = -8 + 2$$

$$\Rightarrow 4 + 2k = -6 \Rightarrow 2k = -10 \Rightarrow k = -5$$

$$\boxed{k = -5}$$

(18) $\lim_{x \rightarrow 3} \frac{(\sqrt{x+1} - 2)}{(x-3)} = \lim_{x \rightarrow 3} \frac{(\sqrt{x+1} - 2)(\sqrt{x+1} + 2)}{(x-3)(\sqrt{x+1} + 2)}$

(7 pts)

$$= \lim_{x \rightarrow 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x \rightarrow 3} \frac{(x-3)}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x \rightarrow 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{(3)+1}+2} = \boxed{\frac{1}{4}}$$

(19) $-\cos(x)$ continuous everywhere

(7 pts) $\tan(x)$ DISCONTINUOUS at $x = \frac{\pi}{2} \neq x = \frac{3\pi}{2}$

must check at $x=0$

(i) $f(0) \stackrel{\text{TOP}}{=} -\cos(0) = -1$

(ii) $\lim_{x \rightarrow 0} f(x) @ \lim_{x \rightarrow 0^-} f(x) \stackrel{\text{TOP}}{=} \lim_{x \rightarrow 0^-} (-\cos(x)) = -\cos(0) = -1$ $\lim_{x \rightarrow 0^+} f(x) \stackrel{\text{TOP}}{=} \lim_{x \rightarrow 0^+} (-\cos(x)) = -\cos(0) = -1$ $\Rightarrow \lim_{x \rightarrow 0} f(x) = -1$ D.N.E

(b) $\lim_{x \rightarrow 0^+} f(x) \stackrel{\text{BUT}}{=} \lim_{x \rightarrow 0^+} (\tan(x)) = \tan(0) = 0$ $\Rightarrow$ NOT CONT. $\boxed{0, \frac{\pi}{2}, \frac{3\pi}{2}}$ Discont. at $x=0$

(20) $\lim_{x \rightarrow 4} \frac{x^2 - 6x + 8}{x^2 - x - 12} = \lim_{x \rightarrow 4} \frac{(x-4)(x-2)}{(x-4)(x+3)}$
 $= \lim_{x \rightarrow 4} \frac{(x-2)}{(x+3)} = \frac{(4)-2}{(4)+3} = \boxed{\frac{2}{7}}$

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(21) $\lim_{x \rightarrow 0} \frac{2 \tan(3x)}{x} = \lim_{x \rightarrow 0} \frac{2 \sin(3x)}{x \cos(3x)}$
 $= \lim_{x \rightarrow 0} \frac{(2)(3) \sin(3x)}{\cos(3x) (3x)}$
 $= \lim_{x \rightarrow 0} \frac{(2)(3)}{\cos(3(0))} \cdot \lim_{x \rightarrow 0} \frac{\sin(3x)}{(3x)} \xrightarrow{1}$
 $= \frac{(2)(3)}{\cos(0)} \cdot 1 = \frac{6}{1} \cdot 1 = \boxed{6}$

(22) (i) $f_{(-1)} \stackrel{\text{TOP}}{=} 4(-1) - (-1)^2 = -4 - 1 = -5 \checkmark$

(ii) $\lim_{x \rightarrow 1} f(x) \stackrel{\text{TOP}}{=} \lim_{x \rightarrow (-1)^-} (4x - x^2) = 4(-1) - (-1)^2 = -5 \checkmark$

(b) $\lim_{x \rightarrow (-1)^+} f(x) \stackrel{\text{BOT}}{=} \lim_{x \rightarrow (-1)^+} \left(\frac{15}{(-1)+4} \right) = \frac{15}{3} = 5 \checkmark$

so $\lim_{x \rightarrow (-1)} f(x)$ D.N.E

so

$f(x)$ IS NOT continuous at $x = -1$

(23) Prove $\lim_{x \rightarrow -3} (-5x-6) = 9$

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(I) Let $\epsilon > 0$.

(II) Search for δ

$$|f(x) - L| < \epsilon \Rightarrow |(-5x-6) - 9| < \epsilon$$

$$\Rightarrow |-5x-15| < \epsilon$$

$$\Rightarrow |-5||x+3| < \epsilon$$

$$\Rightarrow |x - (-3)| < \frac{\epsilon}{|-5|} \quad \text{choose } \delta = \frac{\epsilon}{|-5|}$$

(III) Verify this δ :

$$0 < |x - a| < \delta \Rightarrow |x - (-3)| < \frac{\epsilon}{|-5|}$$

$$\Rightarrow |-5||x+3| < \epsilon$$

$$\Rightarrow |-5(x+3)| < \epsilon$$

$$\Rightarrow |-5x-15| < \epsilon$$

$$\Rightarrow |-5x-6+6-15| < \epsilon$$

$$\Rightarrow |(-5x-6) - 9| < \epsilon$$

$$\Rightarrow |f(x) - L| < \epsilon$$