CALC I EXAM I SOLUTIONS

I

FORM (B)

1. C   search for δ: $|f(x) - L| < \epsilon \Rightarrow |(-6x-1) - (-13)| < \epsilon \Rightarrow$

$\Rightarrow |-6x-1+13| < \epsilon \Rightarrow |-6x+12| < \epsilon \Rightarrow |-6||x-2| < \epsilon \Rightarrow$

$\Rightarrow |x-2| < \frac{\epsilon}{|-6|}$ choose $\delta = \frac{\epsilon}{|-6|}$

2. E $2x-7$ is a polynomial so no discontinuities if $x<5$

$\frac{-8}{x-7} \Rightarrow x-7=0 \Rightarrow x=7$ one discontinuity so far.

must check $x=5$ (i) $f(5) \overset{TOP}{=} 2(5)-7 = 3$ ← (ii) $\lim_{x\to 5^-} f(x)$

(a) $\lim_{x\to 5^-} f(x) \overset{TOP}{=} \lim_{x\to 5^-} (2x-7) = 2(5)-7 = 3$ ←

(b) $\lim_{x\to 5^+} f(x) \overset{BUT}{=} \lim_{x\to 5^+} \frac{-8}{x-7} = \frac{-8}{5-7} = 4$ ←

so $\lim_{x\to 5} f(x)$ D.N.E $\Rightarrow$ discontinuity at $x=5$

so there are 2 total discontinuities.

3. D   The unrestricted limit $= 8$ forces Both one-sided limits to agree and equal 8.

4. B   For some reason I included the same question twice. (see #2)

5. C

(6). C $\lim_{x\to\pi} \tan(x) = \tan(\pi) = 0$ SO NOT B

$\lim_{x\to 1} \frac{(x-2)}{(x^2-4)} = \frac{(1)-2}{(1)^2-4} = \frac{-1}{-3} = \frac{1}{3}$ SO NOT D

$\lim_{x\to(-3)} \frac{(x^2-9)}{(x+3)} \Rightarrow \frac{(-3)^2-9}{(-3)+3} \to \frac{0}{0} \to$ REQUIRES MORE WORK.

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(7) C $\lim_{x\to 4^+} f(x) \overset{BOT}{=} \lim_{x\to 4^+} (2x-5) = 2(4)-5 = 3$

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(8). E NOT $x=6$ because that is not in the region $x \le 4$

yes at $x=5$. $\left.\begin{aligned} \lim_{x\to 4^-}\left(2-\frac{16}{(4-6)^2}\right) &= 2-\frac{16}{4} = -2 \\ \lim_{x\to 4^+}\left(2(4)+\frac{10}{(4)-5}\right) &= 8+(-10) = -2 \end{aligned}\right\}$ NOT at $\underline{x=4}$

so the only discontinuity is at $x=5$

(9) C Discontinuous if $\cos(x)=0 \Rightarrow$ odd multiples of $\pi/2$

$\Rightarrow (2k+1)\pi/2$

(10) A continuity requires all these to exist at $x=-3$

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(11) B $0<|x-a|<\delta \Rightarrow 0<|x-a| \Rightarrow x \ne a$.

(12). E $\lim_{x\to\pi/2^-} f(x) \overset{Top}{=} \lim_{x\to\pi/2^-} (1+2x-\tan(2x)) = 1+2(\frac{\pi}{2}) - \tan(2\frac{\pi}{2})$

$= 1+\pi + \tan(\pi) = \underline{1+\pi}$

$\lim_{x\to\pi/2^+} f(x) \overset{BOT}{=} \lim_{x\to\pi/2^+} (2x-\cos(2x)) = 2(\pi/2) - \cos(2(\pi/2)) =$

$= \pi - \cos(\pi) = \pi-(-1) = \underline{1+\pi}$

(13). E the value of f at $x=7$ has no influence on the limit values.

(14). B $\lim_{x\to 10} \frac{(\sqrt{x+15}-6)}{(x-9)} = \frac{\sqrt{10+15}-6}{(10-9)} = \frac{\sqrt{25}-6}{1} = \frac{-1}{1} = -1$

(15) C We discussed this exact problem in class.

II (16) $\lim_{x\to a}[kf(x)] = k[\lim_{x\to a} f(x)]$

(17). $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a}[f(x)] \pm \lim_{x\to a}[g(x)]$

(18). $\lim_{x\to a}[k] = k$

(19) (1) $f(a)$ is defined

(2) $\lim_{x\to a} f(x)$ is defined

(3) $\lim_{x\to a} f(x) = f(a)$

III (16) (7pts)

$x^2-4=0$

$\Rightarrow (x+2)(x-2)=0$

$\Rightarrow \underline{x=-2}$ $\cancel{x=2}$

: not in the restricted interval $x \leq 1$

$3-x^2$ is a polynomial $\Rightarrow$ none here

Must check $x=1$

(i) $f(1) \overset{\text{TOP}}{=} \frac{-3}{(1)^2-4} = \frac{-3}{-3} = 1$

(ii) $\lim_{x\to 1} f(x)$ (a) $\lim_{x\to 1^-}\left(\frac{-3}{(1)^2-4}\right) = 1$

(b) $\lim_{x\to 1^+}(3-(1)^2) = 3-1 = 2$

so $\lim_{x\to 1} f(x)$ D.N.E.

Discontinuity at $x=1$

so $\boxed{\text{NOT continuous at } x=1,\ x=-2}$

(17) Continuous if $\lim_{x\to(-2)^-} f(x) = \lim_{x\to(-2)^+} f(x)$

(7 pts)

$$\Rightarrow \lim_{x\to(-2)^-} (x^2 - kx) = \lim_{x\to(-2)^+} (4x+2)$$

$$\Rightarrow (-2)^2 - k(-2) = 4(-2)+2$$

$$\Rightarrow 4 + 2k = -8+2$$

$$\Rightarrow 4+2k = -6 \Longrightarrow 2k = -10 \Rightarrow \boxed{k=-5}$$

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(18) $$\lim_{x\to 3} \frac{(\sqrt{x+1}-2)}{(x-3)} = \lim_{x\to 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$$

(7 pts)

$$= \lim_{x\to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{(3)+1}+2} = \boxed{\frac{1}{4}}$$

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(19) $-\cos(x)$ continuous everywhere

(7 pts) $\tan(x)$ DISCONTINUOUS at $x = \pi/2$ & $x = 3\pi/2$

must check at $x=0$

(i) $f(0) \overset{TOP}{=} -\cos(0) = -1$

(ii) $\lim_{x\to 0} f(x)$ (a) $\lim_{x\to 0^-} f(x) \overset{TOP}{=} \lim_{x\to 0^-} (-\cos(x)) = -\cos(0) = -1$ } $\Rightarrow \lim_{x\to 0} f(x)$ D.N.E.

(b) $\lim_{x\to 0^+} f(x) \overset{BOT}{=} \lim_{x\to 0^+} (\tan(x)) = \tan(0) = 0$ — Discont. at $x=0$

SO NOT CONT: $\boxed{0, \pi/2, 3\pi/2}$

20. $\lim_{x\to 4} \dfrac{x^2-6x+8}{x^2-x-12} = \lim_{x\to 4} \dfrac{\cancel{(x-4)}(x-2)}{\cancel{(x-4)}(x+3)}$

$= \lim_{x\to 4} \dfrac{(x-2)}{(x+3)} = \dfrac{(4)-2}{(4)+3} = \boxed{\dfrac{2}{7}}$

21. $\lim_{x\to 0} \dfrac{2\tan(3x)}{x} = \lim_{x\to 0} \dfrac{2\sin(3x)}{x\cos(3x)}$

$= \lim_{x\to 0} \dfrac{(2)(3)\sin(3x)}{\cos(3x)\,(3x)}$

$= \lim_{x\to 0} \dfrac{(2)(3)}{\cos(3(0))} \cdot \lim_{x\to 0} \dfrac{\sin(3x)}{(3x)} \rightarrow 1$

$= \dfrac{(2)(3)}{\cos(0)} \cdot 1 = \dfrac{6}{1} \cdot 1 = \boxed{6}$

22. (i) $f(-1) \overset{\text{TOP}}{=} 4(-1) - (-1)^2 = -4 - 1 = -5$

(ii) $\lim_{x\to -1} f(x)$

ⓐ $\lim_{x\to(-1)^-} f(x) \overset{\text{TOP}}{=} \lim_{x\to(-1)^-} (4x - x^2) = 4(-1) - (-1)^2 = -5$

ⓑ $\lim_{x\to(-1)^+} f(x) \overset{\text{BUT}}{=} \lim_{x\to(-1)^+} \left(\dfrac{15}{(-1)+4}\right) = \dfrac{15}{3} = 5$

so $\lim_{x\to(-1)} f(x)$: D.N.E

so $\boxed{f(x) \text{ IS NOT continuous at } x = -1}$

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(23) Prove $\lim_{x \to (-3)} (-5x-6) = 9$

(I) Let $\epsilon > 0$.

(II) Search for δ

$|f(x) - L| < \epsilon \Rightarrow |(-5x-6) - 9| < \epsilon$

$\Rightarrow |-5x-15| < \epsilon$

$\Rightarrow |-5||x+3| < \epsilon$

$\Rightarrow |x-(-3)| < \frac{\epsilon}{|-5|}$ choose $\delta = \frac{\epsilon}{|-5|}$

(III) Verify this δ:

$0 < |x-a| < \delta \Rightarrow |x-(-3)| < \frac{\epsilon}{|-5|}$

$\Rightarrow |-5||x+3| < \epsilon$

$\Rightarrow |-5(x+3)| < \epsilon$

$\Rightarrow |-5x-15| < \epsilon$

$\Rightarrow |-5x-6+6-15| < \epsilon$

$\Rightarrow |(-5x-6)-9| < \epsilon$

$\Rightarrow |f(x) - L| < \epsilon$