SOLUTIONS

I

1 a $\|\vec{v}\| = \|\langle 4,-1\rangle\| = \sqrt{(4)^2+(-1)^2} = \sqrt{16+1} = \boxed{\sqrt{17}}$

b. $10\vec{v} - 3\vec{w} = 10\langle 4,-1\rangle - 3\langle -5,-3\rangle$

$= \langle 40,-10\rangle - \langle -15,-9\rangle$

$= \langle 40-(-15), -10-(-9)\rangle$

$= \langle 40+15, -10+9\rangle = \boxed{\langle 55,-1\rangle}$

c $\|\vec{v}-\vec{w}\| = \|\langle 4,-1\rangle - \langle -5,-3\rangle\|$

$= \|\langle 4-(-5), -1-(-3)\rangle\|$

$= \|\langle 9, 2\rangle\|$

$= \sqrt{(9)^2+(2)^2}$

$= \sqrt{81+4} = \boxed{\sqrt{85}}$

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2 a $\vec{v}\cdot\vec{w} = (6\vec{i}-2\vec{j})\cdot(5\vec{i})$

$= (6)(5)+(-2)(0)$

$= 30+0$

$= \boxed{30}$

b. $\vec{t}\cdot(\vec{v}+\vec{w}) = (3\vec{i}-\vec{j})\cdot\left[(6\vec{i}-2\vec{j})+(5\vec{i})\right]$

$= (3\vec{i}-\vec{j})\cdot\left[(6+5)\vec{i}-2\vec{j}\right]$

$= (3\vec{i}-\vec{j})\cdot(11\vec{i}-2\vec{j})$

$= (3)(11)+(-1)(-2)$

$= 33+2 = \boxed{35}$

(c) $\vec{0} \cdot \vec{w} = \boxed{0}$

$\longrightarrow = (0\vec{i} + 0\vec{j}) \cdot (5\vec{i})$

$= (0)(5) + (0)(0)$

$= \boxed{0}$

(d) $\cos(\theta) = \dfrac{\vec{v} \cdot \vec{t}}{\|\vec{v}\| \cdot \|\vec{t}\|} = \dfrac{(6\vec{i} - 2\vec{j}) \cdot (3\vec{i} - \vec{j})}{\sqrt{(6)^2 + (-2)^2} \cdot \sqrt{(3)^2 + (-1)^2}}$

$= \dfrac{(6)(3) + (-2)(-1)}{\sqrt{36+4} \cdot \sqrt{9+1}}$

$= \dfrac{20}{\sqrt{40} \cdot \sqrt{10}} = \dfrac{20}{\sqrt{400}} = \dfrac{20}{20} = 1$

so $\cos(\theta) = 1$

$\Rightarrow \theta = \cos^{-1}(1) = \boxed{0°}$

(e) $\vec{u}_{\vec{w}} = \dfrac{1}{\|\vec{w}\|} \vec{w} = \dfrac{1}{\sqrt{(5)^2 + (0)^2}} \cdot 5\vec{i} = \dfrac{1}{\sqrt{25}} 5\vec{i} = \dfrac{1}{5} \cdot 5\vec{i}$

$= \boxed{\vec{i}}$

(f) $\vec{w} \cdot \vec{t} = (5\vec{i}) \cdot (3\vec{i} - \vec{j})$

$= (5)(3) + (0)(-1)$

$= 15 + 0 = 15 \neq 0$ $\boxed{\therefore \text{ NOT ORTHOGONAL}}$

(II)

(3) $\begin{vmatrix} 4 & 6 \\ 1 & -5 \end{vmatrix} = (4)(-5) - (6)(1)$

$= -20 - 6$

$= \boxed{-26}$

(4) $\begin{vmatrix} 10 & 3 \\ -5 & -2 \end{vmatrix} = (10)(-2) - (3)(-5)$

$= -20 - (-15)$

$= -20 + 15$

$= \boxed{-5}$

5. $$\begin{vmatrix} 1 & 2 & 3 \\ -2 & 1 & 1 \\ 4 & 1 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}(1) - \begin{vmatrix} -2 & 1 \\ 4 & -2 \end{vmatrix}(2) + \begin{vmatrix} -2 & 1 \\ 4 & 1 \end{vmatrix}(3)$$

$$= \left[(-2)(1) - (1)(1)\right](1) - \left[(-2)(-2) - (1)(4)\right](2) + \left[(-2)(1) - (4)(1)\right](3)$$

$$= (-3)(1) - (0)(2) + (-6)(3)$$

$$= -3 - 0 + -18 = \boxed{-21}$$

III. 5. $$\|\vec{v} - \vec{w}\| = \|(3\vec{i} - \vec{j} + \vec{k}) - (-\vec{i} + 2\vec{j} + \vec{k})\|$$

$$= \|(3-(-1))\vec{i} + (-1-2)\vec{j} + (1-1)\vec{k}\|$$

$$= \|(4\vec{i} - 3\vec{j})\|$$

$$= \sqrt{(4)^2 + (-3)^2 + (0)^2}$$

$$= \sqrt{16+9+0} = \sqrt{25} = \boxed{5}$$

6. $$\vec{v} \cdot \vec{w} = (3\vec{i} - \vec{j} + \vec{k}) \cdot (-\vec{i} + 2\vec{j} + \vec{k})$$

$$= (3)(-1) + (-1)(2) + (1)(1)$$

$$= (-3) + (-2) + 1 = \boxed{-5}$$

7. $$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 1 \\ -1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix}\vec{i} - \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix}\vec{j} + \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix}\vec{k}$$

$$= \left[(-1)(1) - (1)(2)\right]\vec{i} - \left[(3)(1) - (1)(-1)\right]\vec{j} + \left[(3)(2) - (-1)(-1)\right]\vec{k}$$

$$= [-1-2]\vec{i} - [3+1]\vec{j} + [6-1]\vec{k}$$

$$= \boxed{-3\vec{i} - 4\vec{j} + 5\vec{k}}$$

III

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(8) $z = 9 - 8i \to$ in QIV

$r = \sqrt{(9)^2 + (-8)^2}$ $\tan(\theta) = \frac{-8}{9}$

$= \sqrt{81 + 64}$ $\Rightarrow \theta_R = \tan^{-1}\left(\frac{-8}{9}\right)$

$= \sqrt{145}$ $= 41.6$

≈ 12.0 $\Rightarrow \theta = 360 - 41.6 = 318.4$

$\Rightarrow \boxed{z = 12.0 \text{ cis}(318.4)}$

(9) $z = -8 + 6i$ in QII

$r = \sqrt{(-8)^2 + (6)^2}$ $\tan(\theta) = \frac{6}{-8}$

$= \sqrt{64 + 36}$ $\Rightarrow \theta_R = \tan^{-1}\left(\frac{6}{-8}\right)$

$= \sqrt{100}$ $\approx 36.9°$

$= 10$ $\theta = 180 - 36.9 = 143.1$

$\Rightarrow \boxed{z = 10 \text{ cis}(143.1)}$

(10) $z = -5i \Rightarrow$ lies on the negative y-axis.

$r = \sqrt{(-5)^2 + (0)^2}$ since z lies on the negative x-axis

$= \sqrt{25}$ $\Rightarrow \theta = 270$

$= 5$

$\Rightarrow \boxed{z = 5 \text{ cis}(270)}$

V. (11) $z_1 z_2 = (3 \text{ cis}(300))(24 \text{ cis}(120))$

$= (3)(24) \text{ cis}(300 + 120)$

$= 72 \text{ cis}(420)$ $420 - 360 = 60$

$\Rightarrow = \boxed{72 \text{ cis}(60)}$

(12) $\frac{z_3}{z_4} = \frac{(210 \text{ cis}(75))}{15 \text{ cis}(15)} = \frac{210}{15} \text{ cis}(75 - 15)$

$= \boxed{14 \text{ cis}(60)}$

(13) $(z_1)^3 = (3\text{cis}(300))^3$

$= 3^3 \text{cis}(3(300))$

$= 27 \text{ cis}(900)$ $\quad 900 - 720 = 180$

$= \boxed{27 \text{cis}(180)}$

(14) $\sqrt[5]{z_3} = (z_3)^{1/5} = (210 \text{ cis}(75))^{1/5}$

$= (210)^{1/5} \text{cis}(1/5(75))$

$\approx \boxed{2.9 \text{ cis}(15)}$

(VI) (15) $r = 5 - 5\cos(\theta)$ form: $a - b\cos(\theta)$

since $a = b \Rightarrow$ cardioid

since $\cos(\theta) \rightarrow$ Horizontal orientation

since negative, max value is to the left

max distance = 10

side distances = 5

SEE GRAPH PAPER NEXT SHEET

(16) $r = 4\sin(3\theta)$

Rose curve

petal length = 4

petals = 3 (since it is odd)

∡ between petals $= \frac{360}{3} = 120^\circ$

since "sine" does not lie on the polar axis

see graph paper next sheet

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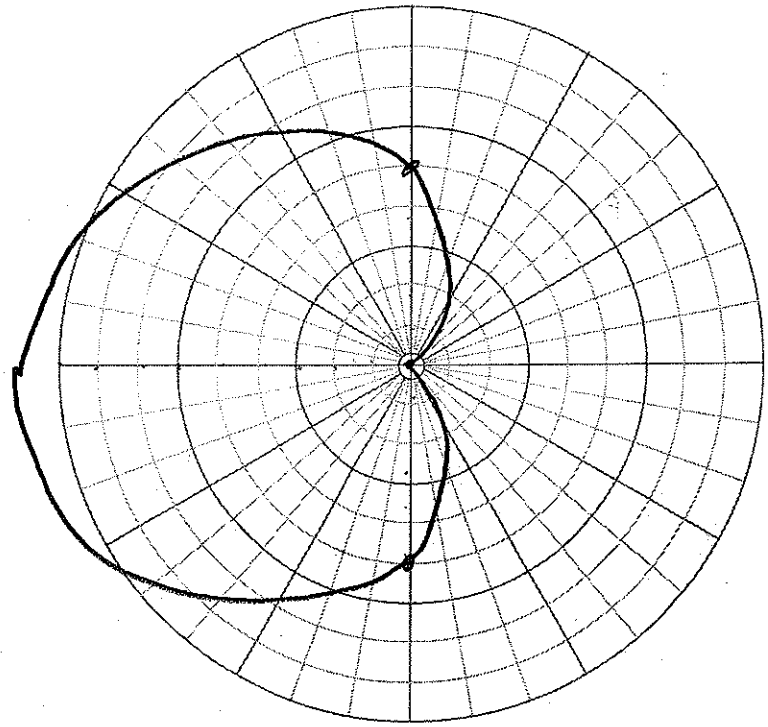

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